The Dissertation Committee for Lizhen Xu
certifies that this is the approved version of the following dissertation:

Economic Analysis of Search Advertising: Price Competition, Bidding Incentive, Consumer Search, and Information Structure

Committee:

Andrew B. Whinston, Supervisor

Anantaram Balakrishnan

P. K. Kannan

John R. Mote

Dale O. Stahl II

Economic Analysis of Search Advertising: Price Competition, Bidding Incentive, Consumer Search, and Information Structure

by

Lizhen Xu, B.E.; M.S.Eco.

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To my parents for their unconditional love and support.

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**Economic Analysis of Search Advertising:
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Information Structure**

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Supervisor: Andrew B. Whinston

This dissertation performs economic analysis of search advertising from a comprehensive picture of the competition facing advertisers—by incorporating the price competition to endogenously investigate advertisers' bidding incentive, and taking into account consumers' online search and the unique information structure associated with the search advertising format. It consists of three essays based on game-theoretic modeling. The first essay studies the oligopolistic price competition among advertisers placed in different advertising positions, considering distinctive features of consumers online search behaviors. We find an interesting *local-competition* pattern in which direct price competition occurs only between advertisers adjacent to each other. The second essay integrates the price competition into the bidding competition and investigates the endogenous bidding incentives of advertisers with different competitive strengths. Surprisingly, we find that an advertising position with a better exposure may not always be profitable for the advertisers with competitive advantage, even if it is cost free. We also show that the bidding outcome might not align with the relative competitive strength. The third essay further considers the effects of organic listing as a competing information source on the sponsored bidding competition and the outcome performances in search advertising. It provides answers to questions such as whether and why advertisers with sufficient exposure from the organic

list may still be willing to bid for top sponsored positions, and how the existence of organic listing affects search engines revenue, consumer surplus, and social welfare.

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Chapter 1

Introduction

Search advertising has been recognized as one of the most effective advertising formats in the Internet era. In search advertising, in response to each user search query (characterized by certain search terms, or "keywords"), search engines return a search engine results page (SERP), which usually consists of two lists: one list of advertisements (with each advertisement composed of a link to the advertiser's website and a short description), called the *sponsored* list, alongside a non-advertising list of general search results, or the *organic* list. Advertisers bid to be placed in these sponsored advertising positions. Such an advertising format proves to be very popular among advertisers, evidenced by its rapid revenue growth. According to a latest industry survey conducted by PricewaterhouseCoopers (PwC) and the Interactive Advertising Bureau (IAB), the Internet advertising revenues in the United States totaled $26.04 billion for 2010, 14.9% higher than 2009. Internet advertising surpassed newspapers to become the second largest advertising medium, next only to television, and is predicted to become the leading advertising medium in just a few years. Within the great revenues generated by Internet advertising, search advertising certainly plays a major role. It accounts for about half of the total Internet advertising revenues, surpasses all of its predecessors such as display banner or email advertising, and continues to grow rapidly.

The huge industry success naturally drives great academic interest. There has been a largely increasing volume of research on search advertising related topics in the fields of information systems, economics, and marketing, including both theoretical studies and empirical investigation. Most of the early theoretical studies focus on the bidding strategies of advertisers and the auction mechanism design of the search engine. These studies typically treat advertisers' valuation (or willingness-to-pay) for those advertising positions as given and assume that the values per click on these ads are exogenously fixed. Nevertheless, in search advertising, the objects being auctioned are advertisements, whose main purpose is to disseminate product information and to promote product sales. In this sense, the values of search advertising positions should be determined *endogenously* within the product market competition. Incorporating product market competition into the search advertising competition thus becomes a crucial step towards a better understanding of advertisers' optimal strategies. Motivated to fill this gap, this dissertation performs economic analysis of search advertising from a comprehensive picture of the competition facing advertisers—by incorporating the price competition to endogenously investigate advertisers' bidding incentive, and taking into account consumers' online search behavior and the unique information structure associated with this particular advertising format. The dissertation consists of three essays, detailed in Chapters 2 through 4, respectively.

In the first essay entitled "Oligopolistic Pricing with Online Search," we start with the price competition alone. We set up a game-theoretic model to examine the oligopolistic price competition among advertisers placed in different advertising positions along a list. We consider two features of consumers' online search: most consumers follow a common search order and some may have negligible search costs. We find that in equilibrium advertisers set their prices probabilistically rather than deterministically, and different advertisers follow different price distributions. The equilibrium pricing pattern exhibits an interesting *local-competition* feature, in which direct price competition occurs only between advertisers adjacent to each other. Further, we incorporate consumers' search strategies into the model so that both the search order and the stopping rules are determined rationally by consumers. We show that similar patterns may continue to hold in the fully rational framework when consumers have higher inspection costs for inferior positions.

With the understanding of pricing under online search, we next integrate price competition into the bidding competition in search advertising and investigate the endogenous bidding incentives of advertisers with different competitive strengths. In the second essay entitled “Price Competition and Endogenous Valuation in Search Advertising,” we consider a game-theoretic model in which firms compete for advertising positions and then compete in price for customers in a product market. Firms differ in their competence, and positions are differentiated in their prominence, which reflects consumers' online search behavior. We find that when endogenously evaluated within the product market competition, a prominent advertising position might not always be desirable for a firm with competitive advantage, even if it is cost-free. The profitability of a prominent advertising position depends on the trade-off between the extra demand from winning the position and the higher equilibrium prices when the weaker competitor wins it. We also show that the bidding outcome might not align with the relative competitive strength, and an advantaged firm might not be able to win the prominent position even when it does value that position. We derive two-dimensional equilibrium price dispersion, with the realized prices at the same position varying and the expected prices differing across different positions. We find that the expected price in the prominent position might not always be higher, implying that an expensive location does not necessarily lead to expensive products.

The first two essays, like most existing studies, focus on sponsored listing alone. Nevertheless, to deepen our understanding of search advertising from a more comprehensive perspective, we should also consider the effects of organic listing. The third essay entitled “Interplay Between Organic Listing and Sponsored Bidding in Search Advertising” aims to explore the effects of organic listing as a competing information source on the advertising competition (i.e., sponsored bidding) and the outcome performances in search advertising. We set up a game-theoretic model in which firms bid for sponsored advertising slots and compete for consumers in the product market. Firms are asymmetrically differentiated in terms of market preference and are placed at organic slots with different prominence based on their relative popularity. We suggest that when facing two competing lists, leading firms' sponsored bidding incentive is mainly *preventive*, whereas small firms' sponsored bidding incentive is mainly *promotive*. We show that these two incentives change in opposite directions when the difference in advertisers' competitive strength decreases. As a result, even small firms may outbid stronger competitors and win good sponsored positions under such a co-listing setting. We further analyze the effects of the organic listing on equilibrium outcomes by comparing it with a benchmark case in which there is only a sponsored list. We find that organic listing compensates the leading firms to help smaller firms win better sponsored positions, which balances the equilibrium information structure through sponsored list without impairing the objectivity of the organic list. While organic listing may lower the search engine's short-term revenue, it increases equilibrium consumer surplus, social welfare and sales diversity, which are in the long-term interest of the search engine. Finally, we suggest some possible direction to improve the performance of organic listing for highly asymmetric markets.

Chapter 2

Oligopolistic Pricing with Online Search

2.1 Introduction

The Internet has greatly improved the efficiency of information sharing. On the one hand, the Internet greatly reduces the physical cost of accessing product information; on the other hand, the rapidly developed ecommerce applications, especially online sponsored advertising, brings merchants selling similar products together and facilitates consumers' searching. Nevertheless, the prediction of "the law of one price" has not been realized in the Internet era. Price dispersion has been well documented and discussed in the information systems literature Chen and Hitt (2002); Clemons et al. (2002); Smith and Brynjolfsson (2001). This work draws upon consumer online search behavior and studies the oligopolistic equilibrium pricing to offer a theoretical explanation of online price dispersion.

One classical view in economic theory attributes price dispersion to the heterogeneity in consumers' search behavior Burdett and Judd (1983); Stahl (1989). These works typically study price competition in traditional offline search markets, and their model settings may not fit the online search very well. Compared to the traditional offline search, two features are more salient in the online environment: First, because of considerable differences in the location or the visibility of business positions (e.g., the hyperlinks), there often exists a common ordering in consumer search. Second, consumers' search costs are highly diversified; in particular, there exists a substantial portion of "shoppers" who have non-positive search costs.

Differences in location and the way of presenting on a webpage, or different virtual positions in the hyperlinking network on the Internet, cause firms' links to differ greatly in terms of their visibility or prominence level. For example, search engines commonly display a limited number of premium sponsored links in a highlighted area with a large size and a bright color on the top of a search result page; they also display a list of regular sponsored links in the right column of a webpage with no highlighting, among which the top positions are commonly believed to be superior to the lower positions. When display space is limited (e.g., on mobile devices), sometimes only one or two ads are displayed at a time, and users have to switch to another page to view additional ads. Because of human eye movement patterns and information processing habits, consumers usually pay different levels of attention to different positions. Experimental studies have shown that consumers pay more attention to the content with colors and a large size Lohse (1997), and compared to paper media, consumers are more likely to focus on ads near the heading of electronic lists Hoque and Lohse (1999). Online statistics also show a significant decrease in traffic from the top sponsored link downward and much fewer visits after the first page Brooks (2004). As a result, most consumers browse links following a common order: they inspect the most prominent positions first, and then some stop while others continue to inspect the less prominent ones. In fact, there are also examples in the physical world that resemble such ordering: most consumers first look at shelf slots at the eye level in a supermarket Dreze et al. (1994) or first visit the storefronts near the main entrance of a shopping mall, and then some of them continue to the floor-level shelf slots or the corner storefronts.

In reality such a common ordering could simply be the direct result of human habits and not necessarily a strategic decision after sophisticated calculation;[1]nevertheless, we can also explain such ordering in a fully rational framework. As we show, the ordered search can be derived as an equilibrium outcome originating from differences in the inspection costs for different positions. In particular, the inferior positions incur higher inspection costs than the superior ones. A higher inspection cost can be interpreted as the psychological

[1]Peterson and Merino (2003) argues that in reality, consumers search in a way different from the rational assumptions in the economic theory, and this situation continues in the online world.

resistance to overcoming the information processing habits, or the extra effort in locating a less prominent link or switching webpages. Given such differences, consumers' choice to to follow a certain order is a rational search strategy in equilibrium.

The second feature of online search behavior owes to the convenience brought by the Internet, which reduces consumers' search costs from driving to the store to making several clicks of the mouse. In addition, it has also been shown that some consumers derive hedonic utility from shopping online Childers et al. (2001). As a result, a certain portion of consumers actually has a non-positive (zero or even negative) net search cost. We call them *shoppers*. However, not everybody shopping online has such time luxury. The convenience of electronic commerce attracts people with time constraints, whose only goal is to find a product within the shortest amount of time. Thus, a certain portion of consumers has positive search costs. They do not conduct an exhaustive search and stop searching at certain stages, usually after sampling only a few sites (e.g., Johnson et al. (2004) empirically shows that online customers tend to search very few sites on average).

To study the equilibrium pricing pattern, we set up a game-theoretic model, capturing the two features of the online search. We consider oligopolistic competition in which multiple firms compete for consumers in a product market. Firms are differentiated in the prominence level of their positions, which are reflected in the ranks in consumers' presumed search sequence. Consumers are differentiated in their search behavior. In particular, a certain portion of consumers has non-positive search costs and conducts a thorough search. We eliminate heterogeneity among firms and consumers in all other dimensions except firms' position and consumers' search behavior to show that the driving forces of the equilibrium pricing pattern are the two distinctive features of the search behavior.

We find an interesting equilibrium pricing pattern when first taking consumers' search strategies as exogenous. The equilibrium exhibits the feature of "local competition," in which firms compete directly with their neighbors along consumers' search order only. We show that in equilibrium, all firms mix their prices over different supports. Overlaps occur only in the supports of two firms adjacent to each other. Hence, there is no direct competition between any two firms that are not next to each other. We further show that behind the local-competition feature lies a global mutual dependence across all firms. We then endogenize consumers' search strategies so that consumers make fully rational decisions on their search order and stopping rules. We show that similar pricing patterns may continue to hold in equilibrium when consumers' inspection costs for different positions are different.

The main contribution of this work to the economic theory on search and pricing lies in that we study the asymmetric mixed-strategy equilibrium pricing in oligopolistic competition. To the best of our knowledge, the local-competition pattern revealed in this study is absent in the previous literature. The investigation on endogenizing the consumer search enriches the studies that explore asymmetric equilibrium pricing with optimal search strategies.

Diamond (1971) raises the famous paradox that when consumers have positive search costs, an endogenous search model leads to a trivial equilibrium in which all firms charge the monopoly price and consumers do not search. Varian (1980) suggests that when there exist consumers who are "informed" of all firms' prices, the equilibrium outcome may involve mixed-strategy pricing, which leads to price dispersion. Stahl (1989) studies a random search model, in which consumers randomly pick a firm to inspect and all firms are symmetric. That paper considers the existence of a portion of "shoppers" who have zero search cost, and derives a symmetric equilibrium pricing provided that all non-shoppers have the same search cost. Weitzman (1979) formulates the optimal search strategies given firms' (asymmetric) price distributions in a general setting. Arbatskaya (2007) is among the very few that study ordered search and price competition. By considering cost distributions atomless at the zero point, that paper focuses on the pure-strategy equilibrium. There are other studies that analyze mixed-strategy equilibrium in duopolistic competition in different contexts. For example, Campbell et al. (2005) considers the effect of shoppers in a symmetric duopolistic setting, and Narasimhan (1988) and Weber and Zheng (2007) consider asymmetric duopolistic mixed strategies. In contrast to these works, we consider the existence of shoppers in the ordered search market, and explicitly derive asymmetric mixed-strategy equilibrium in oligopolistic price competition with both exogenous and endogenous consumer search.

The rest of the chapter is organized as follows. In Section 2.2, we start with a model in which consumers' search behavior is exogenously given. Section 2.3 details the analysis and shows the main results. In Section 2.4, we endogenize the consumer search such that consumers rationally decide what search order to follow and when to stop. We consider both cases of position-invariant and position-dependent inspection costs. We show that the local-competition pattern may arise in the fully rational framework when consumers have higher inspection costs for inferior positions. Section 2.5 concludes the chapter with a discussion on managerial implications.

2.2 Model

There are n (≥ 2) firms selling homogeneous products and competing for consumers in a product market. These firms have the same marginal production cost, which is normalized to zero without loss of generality. A continuum of consumers with unit mass exists in the market. Each consumer has a unit demand of the product and realizes a unit utility from consuming the product. Therefore, consumers will buy the product only if its price does not exceed 1. Firms are identical except for their ranks in the search ordering, and consumers are identical except for their search behavior. By eliminating differentiation in all other dimensions, we are able to show that the distinctive features of consumers' online search behavior alone could drive an interesting price dispersion pattern.

Consumers obtain product information through an information portal with a list of hyperlinks directed to firms' websites where purchases can be made directly. Firms are placed in different positions in the list, which can be viewed as the outcome of a pre-game competition, such as a bidding competition. Because all firms are identical *ex ante*, the location competition outcome is irrelevant for analyzing the price competition. Any assignment of positions becomes identical after relabeling firms by their position rank. Therefore, we do not include the location competition in the model but start from *after* firms get placed at different positions. Different positions have different prominence levels, which can be strictly ordered. Without loss of generality, we call the most prominent position the *first* position, the second most prominent position the *second* one, and so on. For convenience, we call the firm at the ith position *firm* i ($i = 1, ..., n$).

We start with the case in which consumers' search strategies are exogenously given, in a way reflecting the two unique features of online search patterns: First, there exists a common search ordering so that all consumers start searching from the first position and may continue to the second, then the third, and so forth. Second, consumers' search costs are highly diversified so that they may stop searching at different stages. Especially, there exists a certain portion of *shoppers* who have non-positive search cost and sample all positions before making the purchase decision. Specifically, we assume that after sampling the ith position, a portion of α_i ($0 < \alpha_i < 1$) stops searching, while the other $1 - \alpha_i$ continue to sample the next position. Therefore, the portion of consumers who visit the ith ($i \geq 2$) position is $\Pi_{j=1}^{i-1}(1 - \alpha_j)$. To simplify notation, we denote $\beta_i = \Pi_{j=1}^{i-1}(1 - \alpha_j)$, and let $\beta_1 = 1$ and $\alpha_0 = 0$. To rule out violent fluctuation in the attention declining rates α_i's, we assume that $\alpha_i \geq \alpha_{i+1}(1 - \alpha_i)$ ($1 \leq i < n$). This condition requires that the rate of decline in attention (i.e., α_i's) does not increase dramatically from one position to the next, which can be easily satisfied (e.g., the same rate of decline across positions ($\alpha_i = \alpha_{i+1}$) satisfies this condition).[2]

The timing of the game is as follows. Firms at different positions price their products simultaneously. Consumers sample the position(s), learn the price(s), and make the purchase decision. For those who sample at least two positions, they purchase from the firm with the lowest price. When there is a tie in the lowest price, they randomly pick one of

[2]When the condition does not hold, which implies that a lower position retains a larger portion of the visitors than an upper one, the firm at the upper position may deviate to a lower price range.

the firms, with equal probability.

2.3 Equilibrium with Exogenous Search

We first derive firms' equilibrium pricing strategies and then analyze the pattern of equilibrium price dispersion. In deriving the equilibrium pricing, we notice that any static pricing is unstable due to the existence of shoppers.

Lemma 2.1. (Lack of Pure-Strategy Equilibrium) *There is no pure-strategy equilibrium in the price competition.*

Proof. We prove the above result by contradiction. Suppose there is a pure-strategy pricing equilibrium. If there is no tie in the lowest price, then it is profitable for the firm with the lowest price to deviate by increasing its price closer to but still lower than the second-lowest price. If there is a tie in the lowest price that is strictly positive, then any of the firms with the lowest price has profitable deviation by slightly cutting the price. If there is a tie in the lowest price that is zero, then among all these firms with zero price, the one with the highest prominence level can achieve positive profit by slightly increasing its price. Therefore, pure-strategy pricing cannot be an equilibrium.[3] □

Since there exists a certain portion of consumers who sample all positions to look for the lowest price, a slight cut in price to offer the lowest can lead to a significant increase in market share by capturing this portion of consumers. As a result, competing firms tend to lower their prices relative to the rivals. However, once the price is pushed to a certain low level, the firm at a better position in terms of prominence can be better off by charging a higher price and exploiting those consumers who stop searching right there. Therefore, any pricing strategy in which firms statically charge one price cannot be stable. Clearly, the driving force here is the presence of shoppers and the locational asymmetry created by the search ordering.

[3] The arguments apply to cases both when money is infinitely divisible and when there is a finite money increment, as long as it is sufficiently small.

We next examine the mixed-strategy pricing equilibrium. We use $F_i(p)$, $i = 1, ..., n$, to describe firm i's mixed strategy of pricing. Like regular cumulative distribution functions, $F_i(p)$ measures the probability that firm i charges a price less than or equal to p.

Proposition 2.1. (Equilibrium Pricing and Local-Competition Pattern) *When $\alpha_i \geq \alpha_{i+1}(1-\alpha_i)$ for $1 \leq i < n$, the equilibrium mixed strategy of pricing from position i is as follows:*

$$\begin{aligned} F_i(p) &= \begin{cases} 1 - \frac{\bar{p}_{i+1}}{p} & p \in [\bar{p}_{i+1}, \bar{p}_i) \\ 1 - \frac{\alpha_{i-1}(\bar{p}_{i-1}-p)}{\alpha_i(1-\alpha_{i-1})p} & p \in [\bar{p}_i, \bar{p}_{i-1}] \end{cases} \quad (1 \leq i \leq n-1), \\ F_n(p) &= 1 - \frac{\alpha_{n-1}(\bar{p}_{n-1}-p)}{(1-\alpha_{n-1})p} \qquad p \in [\bar{p}_n, \bar{p}_{n-1}], \end{aligned} \tag{2.1}$$

where $\bar{p}_i$'s are recursively defined as

$$\begin{aligned} &\bar{p}_0 = \bar{p}_1 = 1, \\ &\bar{p}_i = k_{i-1}\bar{p}_{i-1} \quad (i = 2, .., n), \end{aligned} \tag{2.2}$$

and the coefficients k_i's are recursively defined as

$$\begin{aligned} &k_{n-1} = \alpha_{n-1}, \\ &k_i = \frac{\alpha_i}{\alpha_i + \alpha_{i+1}(1-\alpha_i)k_{i+1}} \quad (i = n-2, ..., 1). \end{aligned} \tag{2.3}$$

Proof. All proofs are presented in the Appendix, unless indicated otherwise. □

We next use an example to illustrate the pattern of the equilibrium mixed-strategy pricing.

Example 2.1. *Consider a case of four positions with the same declining rate. Specifically, $n = 4$ and $\alpha_1 = \alpha_2 = \alpha_3 = 1/2$. According to the above recursive definition in Equation (2.3), $k_3 = \alpha_3 = 1/2$, and it can then be derived that $k_2 = 4/5$ and, further, that $k_1 = 5/7$. Thus, according to Equation (2.2), $\bar{p}_1 = 1$, $\bar{p}_2 = k_1\bar{p}_1 = 5/7$, $\bar{p}_3 = k_2\bar{p}_2 = 4/7$, and $\bar{p}_4 = k_3\bar{p}_3 = 2/7$. Notice that by definition, the sequence of price support bounds$\{\bar{p}_i\}_{i=1}^n$ is monotonically decreasing, so that $\bar{p}_1 > \bar{p}_2 > \bar{p}_3 > \bar{p}_4$. The pricing strategies of the four* *firms are as follows:*

$$\begin{aligned}
F_1(p) &= \begin{cases} 1-\frac{5}{7p} & p\in[\frac{5}{7},1) \\ 1 & p=1 \end{cases}, \\
F_2(p) &= \begin{cases} 1-\frac{4}{7p} & p\in[\frac{4}{7},\frac{5}{7}) \\ 1-\frac{2(1-p)}{p} & p\in[\frac{5}{7},1] \end{cases}, \\
F_3(p) &= \begin{cases} 1-\frac{2}{7p} & p\in[\frac{2}{7},\frac{4}{7}) \\ 1-\frac{2(5-7p)}{7p} & p\in[\frac{4}{7},\frac{5}{7}] \end{cases}, \\
F_4(p) &= 1-\frac{4-7p}{7p} \qquad\qquad p\in[\frac{2}{7},\frac{4}{7}].
\end{aligned} \tag{2.4}$$

Figure 2.1 illustrates the supports and distributions for the pricing strategies of the firms in these four positions.

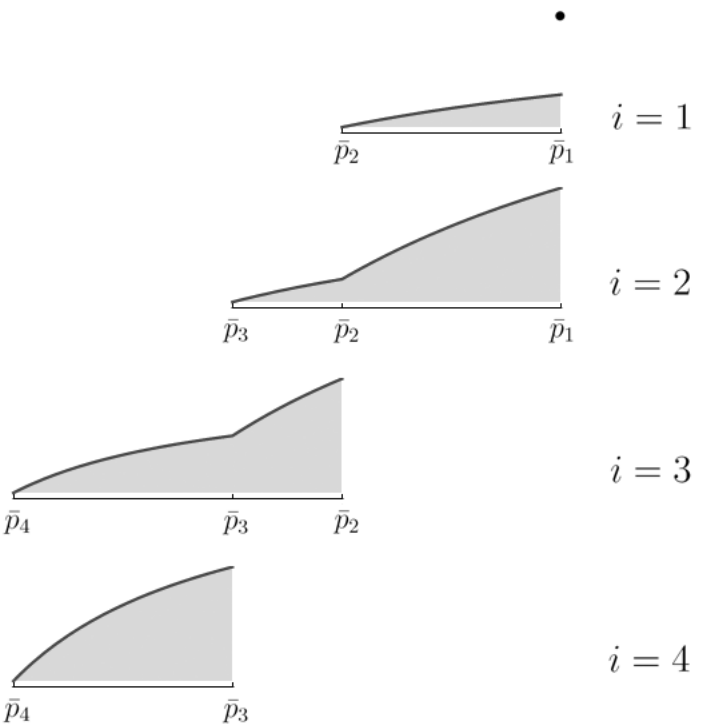
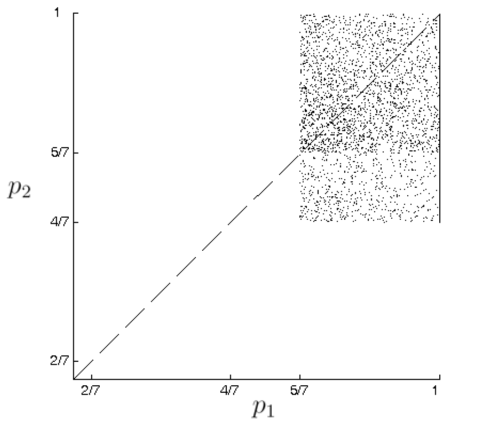
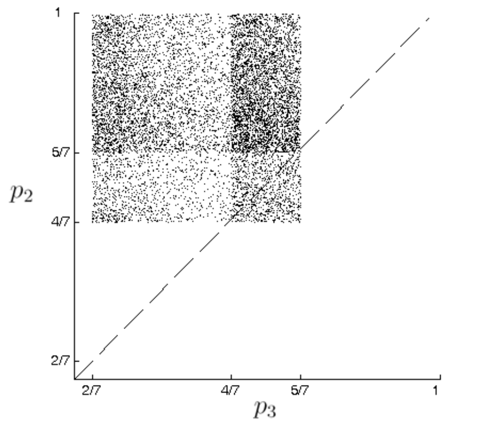


Figure 2.1: Price Supports and Cumulative Distributions for Different Positions

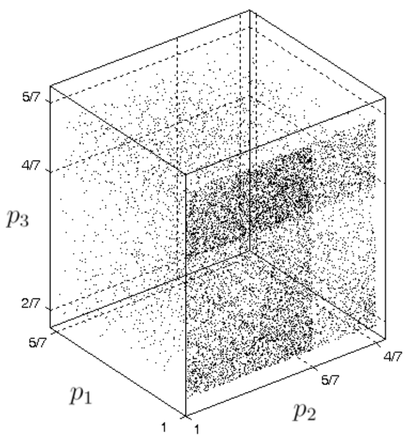
Figure 2.2 depicts the simulated results of the equilibrium in Example 2.1 with each point representing an independent draw from the price distributions in Equation 2.4. Notice that the dotted square areas which the diagonal passes through in (a), (b) and (d) indicate the direct price competition between two adjacent firms.



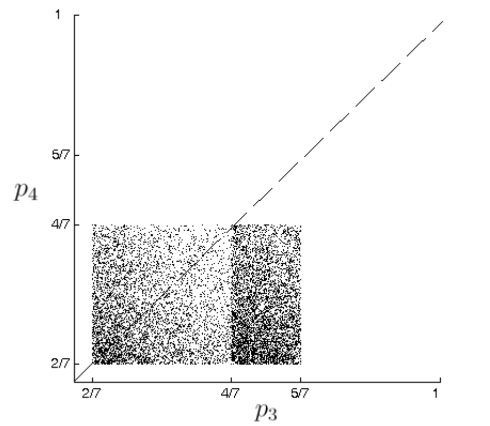
(a) Simulated Prices at Positions 1 and 2



(b) Simulated Prices at Positions 2 and 3



(c) Simulated Prices at Positions 1, 2 and 3



(d) Simulated Prices at Positions 3 and 4

Figure 2.2: Simulation of the Equilibrium in Example 2.1 (10,000 Simulated Points)

In equilibrium, each firm achieves a constant expected payoff by charging any price within its price support. On the one hand, the constant payoff makes each firm willing to randomize price over its entire support. On the other hand, each firm randomizes its price in a way that gives its competitors a constant payoff within their price supports. Specifically, firm i's expected profit can be written as

$$\begin{cases} \pi_i(p) = p\alpha_i\beta_i + p\alpha_{i+1}\beta_{i+1}[1 - F_{i+1}(p)] & p \in [\bar{p}_{i+1}, \bar{p}_i) \\ \pi_i(p) = p\alpha_i\beta_i[1 - F_{i-1}(p)] & p \in [\bar{p}_i, \bar{p}_{i-1}]. \end{cases} \tag{2.5}$$

When firm i $(2 \leq i \leq n-2)$ charges a price within the lower half of its price support $[\bar{p}_{i+1}, \bar{p}_i)$, the firm captures all consumers who sample its site and stop searching there (i.e., a portion of $\alpha_i\beta_i$). This is because its price is lower than firm $(i-1)$'s price for sure (recall that firm $(i-1)$'s price support is $[\bar{p}_i, \bar{p}_{i-2}]$), which accounts for the first term of the right-hand side of the first equation in (4.4). Firm i can capture those who continue to sample the $(i+1)$th position and stop searching there (i.e., a portion of $\alpha_{i+1}\beta_{i+1}$) only when its price is lower than firm $(i+1)$'s price, which accounts for the second term. Naturally, firm i forgoes all those consumers who continue to sample the $(i+2)$th position, since firm $(i+2)$'s price support is $[\bar{p}_{i+3}, \bar{p}_{i+1}]$ and its price is lower for sure. The second equation in (4.4) is the expected profit when firm i prices within the upper half of its price support, which can be interpreted in a similar way. Substituting in the firms' equilibrium pricing strategies and by simple algebra, we can show that by charging any price within its strategy, firm i achieves a constant expected profit $\alpha_i\beta_i\bar{p}_i$.

The most interesting feature of the equilibrium pricing is the *local-competition* pattern; that is, firms compete directly with their neighbors only. There is no overlap between the price supports of any two firms more than one position apart and thus no direct price competition between firms "distant" from each other. The driving forces of such a pattern are the two features of consumers' search behavior. Because of the existence of shoppers, firms do not statically charge one single price but have to mix their prices to compete for consumers. Nevertheless, such competition is localized because of the decrease in visits along the common search order. The firm at a lower position cannot be better off by entering a higher price range, because it would then lose its already quite limited customer base. The firm at a higher position will not undercut its price because in doing so, it would only entangle itself in a fiercer price competition against the lower-ranked firms, which would result in little gain in extra demand but significant loss in the profit from the captured demand. As a result, to compete locally is the equilibrium outcome.

Another interesting aspect of the local-competition pattern is that each firm's equilibrium pricing strategy only involves "local" information. According to Equation (2.1), firm i's price distribution F_i only contains consumer search parameters α_{i-1}, α_i and α_{i+1}. Also, firm i's price support is determined by (the lower bound of) firm $(i-2)$'s support and (the upper bound of) firm $(i+2)$'s support; within its support, firm i's profit is determined only by firm $(i+1)$'s and firm $(i-1)$'s pricing strategies, according to Equation (4.4). In this sense, although the formal equilibrium analysis needs to be based on the whole picture of the game, to formulate optimal pricing strategy in practice, decision makers can simply focus on the traffic information at the adjacent positions and the pricing strategies of the neiboring firms (i.e., firms that are adjacent and one position apart).

It is worth noting that the local-competition pattern is the result of global consideration. Although no direct price competition is explicitly observed in equilibrium, even firms distantly above or below have an impact on a particular firm's pricing strategies. In fact, although firm $i+k$ $(i-k)$ does not compete directly with firm i, it affects firm i's pricing through a chain effect, from firm $i+k-1$ $(i-k+1)$ through firm $i+1$ $(i-1)$. To see this, reconsider Example 2.1. When the fourth firm is eliminated, for example, although the first two firms still have the same neighbors as before, the price support of the first firm shifts toward the right from $[5/7, 1]$ to $[4/5, 1]$, and the second firm's support shifts to the left from $[4/7, 1]$ to $[2/5, 1]$. In fact, when the competitor from below disappears, the third firm tends to increase its price (and shifts its price support from $[2/7, 5/7]$ to $[2/5, 4/5]$). In response, the second firm lowers its price support to capture more demand. Meanwhile, the more intense competition between the second and the third firms drives away the first firm's interest, which leads to the increase of the lower bound of its price support. As we can see, behind the local-competition phenomenon actually lies the global mutual dependence among all firms.

Several other features of equilibrium pricing are also worth noting. First, except the first one, all firms' equilibrium pricing strategies are atomless within their entire supports, including the upper and lower bounds. This is because a mass point in one firm's price distribution would result in a downward jump of another firm's expected demand at that point and, consequently, lower profit levels in a contiguous region right to that point. For this reason, the only possible place where a mass point may occur is the common upper bound of the first two firms' price supports $\bar{p}_1$: although the mass point in $F_1(\cdot)$ causes a downward jump in firm 2's expected profit at $p = \bar{p}_1$, firm 2's actual expected profit is not affected because $F_2(\cdot)$ places a non-positive probability measure on that particular point. This feature under our oligopolistic model is in line with the results derived from duopolistic competition in other settings Kreps and Scheinkman (1983).

The kinks in firms' pricing distributions can be explained by the localized competition. The pricing distributions in the first and second part of the support for firm i ($2 \leq i \leq n-1$) are determined by the competition against its direct neighbors, firms $i-1$ and $i+1$, respectively. As the competition against the firm above and the firm below are generally different, naturally, a kink arises in firm i's pricing distribution at $\bar{p}_i$. The shape of pricing distributions for the first and last firms is distinctive from the rest because those two firms have one direct neighbor only.

The next corollary reveals the monotonic decrease of the expected profit of firms at different positions, which explains why the top position of a sponsored list in online search advertising is usually the most popular and engenders fierce bidding competition.

Corollary 2.1. (Decrease of Expected Profits) *The firms' equilibrium expected profits decrease monotonically from the first toward the last; that is, $\pi_i > \pi_{i+1}$, $i = 1, ..., n-1$.*

Proof. Notice that $\pi_i = \alpha_i \beta_i \bar{p}_i$. Since $\bar{p}_i > \bar{p}_{i+1}$ and $\alpha_i \geq \alpha_{i+1}(1-\alpha_i)$, $\pi_i > \pi_{i+1}$. □

The above corollary shows that location advantage is rewarding in the sense that the firm in the advantageous location earns a higher profit. It is worth pointing out that the profit difference between a higher-ranked position and a lower-ranked one should dissipate in the pre-game location competition if all firms are ex ante identical. That is, a firm has to pay a higher price for a superior position, which counterbalances its profit advantage. There is a rich literature on the competition for better locations or exposure, from the classical advertising literature Robert and Stahl (1993) to the recent work on online advertising and position auctions Weber and Zheng (2007).

The next corollary indicates that the equilibrium pricing under the ordered search with shoppers exhibits two levels of dispersion: not only are the realized prices at different positions different, but the expected prices are also different across positions.

Corollary 2.2. (Decrease of Expected Prices) *The expected price decreases monotonically from the first position toward the last one; that is, $E(p_i) > E(p_{i+1})$, $i = 1, ..., n-1$.*

Notice that firms adjacent to each other adopt similar pricing strategies over the overlapped interval of their supports. In fact, according to Equation (2.4), the conditional probability density functions of their pricing strategies over the overlapped interval are the same, which implies the same conditional expectations; that is, $E(p_i | p \in [\bar{p}_{i+1}, \bar{p}_i]) = E(p_{i+1} | p \in [\bar{p}_{i+1}, \bar{p}_i])$. Because firm i prices within the upper half of its price support $[\bar{p}_i, \bar{p}_{i-1}]$ with positive probability and firm $i+1$ prices within the lower half of the support $[\bar{p}_{i+2}, \bar{p}_{i+1}]$ with positive probability, the unconditional expectation of firm i's price is strictly higher than that of firm $(i+1)$'s.

Corollary 2.2 shows that the equilibrium price expectation decreases monotonically along the direction of consumers' search ordering. As a result, search is rewarding in the sense that those who keep searching are more likely to find a lower price.

2.4 Equilibrium with Endogenous Search Order

In the previous analysis, we take consumers' search behavior, including the search order and stopping rules, as exogenously given. In this section, we extend the analysis to endogenize consumers' search strategies. The focus is to explore whether and under what conditions a similar equilibrium pricing pattern continues to arise in the fully rational framework. As we show, the inherent difference among positions is necessary for the local-competition pattern to arise in equilibrium pricing. When consumers are free to sample any position with no particular ordering constraint and inspecting different positions incurs the same cost, such a pattern disappears. The pattern arises only if the more prominent position incurs a lower sampling cost than the less prominent position, which reflects the underlying distinction between ordered search and random search. In this case, we further show that under certain parametric conditions, equilibrium pricing with the same local-competition pattern can be derived in the fully rational framework.

We now consider consumers as active players in the game. We assume that all consumers are fully rational and decide their search order and stopping rules strategically. We consider the *rational-expectations equilibrium (REE)*, in which consumers' search strategies are rational given firms' pricing strategies and firms have no profitable deviation in pricing given consumers' search strategies. For shoppers with zero search cost, it is always optimal for them to sample all positions before making a purchase decision. For non-shoppers, we consider a sequential search process: the consumer inspects one position and learns the price, and then he or she decides whether to continue searching or to stop and, if to continue, which position to inspect next. In other words, an individual consumer's search strategy consists of a sequence of decisions; each decision $d(z, \mathcal{C})$ can be to stop, to inspect a position, or to randomly inspect several positions with a probability distribution; $d(z, \mathcal{C})$ depends on the lowest price z from all the inspected positions, and the choice set $\mathcal{C}$, which contains all the uninspected positions. To determine $d(z, \mathcal{C})$, the consumer needs to calculate the net expected gain from all possibilities for the next step. The net expected gain from inspecting the ith position, given the lowest sampled price

z and the choice set $\mathcal{C}$, $EG(i; z, \mathcal{C})$, equals the expected decrease in purchase price plus the net expected gain from another rational search afterwards, minus the inspection cost. Notice that when no position has been inspected yet, let $z = 1$, and we thus only need to consider $z \leq 1$ throughout the rest of the chapter. Formally, for an individual consumer with positive inspection costs k_i, given firms' pricing strategies F_i, similar to Weitzman (1979), the net expected gain can be formulated recursively as

$$EG(i; z, \mathcal{C}) = \int_{\underline{p}}^{z} (z - p) \, dF_i(p) - k_i + \int_{\underline{p}}^{\bar{p}} EG^* (\min\{z, p\}, \mathcal{C} \backslash \{i\}) \, dF_i(p), \tag{2.6}$$

where

$$EG^*(z, \mathcal{C}) = \max_{j \in \mathcal{C}} \{EG(j; z, \mathcal{C}), 0\}.$$

To determine the rational search decision is to compare the net expected gain of all the options in the choice set. If further search yields no positive net expected gain, stopping is the rational decision and $EG^* = 0$. Otherwise, the rational search decision is to continue to inspect the position that generates the highest net expected gain. In the case of a tie, randomly inspecting any of them is rational. We next use an example to illustrate consumers' rational search strategy.

Example 2.2. *Consider two firms. Firm 1 sets its price equal to* 1 *or* 0 *with equal probability. Firm 2 prices uniformly over* $[0, \frac{1}{2}]$. *Consumers' inspection costs are the same for both firms, and let* $k_1 = k_2 = \frac{1}{8}$. *The rational search strategy in this case is to inspect the first firm at first: if the quoted price is zero, stop searching; otherwise, continue to inspect the second firm.*

In this example, the expected price of the first firm (1/2) is *higher* than the expected price of the second firm (1/4). This example shows that it may not be rational to start searching from the position with a lower expected price even if the inspection costs are the same. In fact, consumers' rational search strategies generally depend on the full distributions of equilibrium prices, which in turn are determined by consumers' rational search strategies. Such interdependence makes the analysis complex, especially when we consider asymmetric oligopolistic competition and heterogeneous consumer search costs. For this reason, equilibrium analysis of oligopolistic pricing with rational search generally results in no closed-form solution. In this section, we seek to derive explicit equilibrium under certain conditions.

We consider the simplest oligopolistic case of three firms. The three firms are located in three different positions. Again, we refer to the firm located in the ith position as the ith firm or firm i, $i \in \{1, 2, 3\}$. Consumers are different in their search costs. To be consistent with the previous settings, we assume that among all consumers with total mass 1, α_1 of them have the highest search costs and are referred to as type-1 consumers; $\alpha_2 (1 - \alpha_1)$ of them have lower search costs and are referred to as type-2 consumers; the rest $(1 - \alpha_1)(1 - \alpha_2)$ are shoppers. For simplicity, we let $\alpha_1 = \alpha_2 = \alpha$ $(0 < \alpha < 1)$. Assume type-1 consumers incur a cost k_i to inspect the ith position, while type-2 consumers incur a cost $k_i^{'}$, $i \in \{1, 2, 3\}$, $0 \leq k_i^{'} \leq k_i \leq 1$. All other settings follow the previous model setup.

We first study the case in which consumers' inspection costs are position-invariant; that is, $k_1 = k_2 = k_3$ and $k_1^{'} = k_2^{'} = k_3^{'}$. In this case, there is essentially no difference among positions, and firms thus are symmetric. We derive symmetric equilibrium under certain conditions and uncover an interesting equilibrium pricing pattern that involves segmentation of price supports. We also show that the local-competition pattern from the previous analysis does not hold in the case of position-invariant inspection costs. We then allow inspection costs to vary across different positions and give a necessary condition and a sufficient condition for the local-competition pattern to arise in equilibrium.

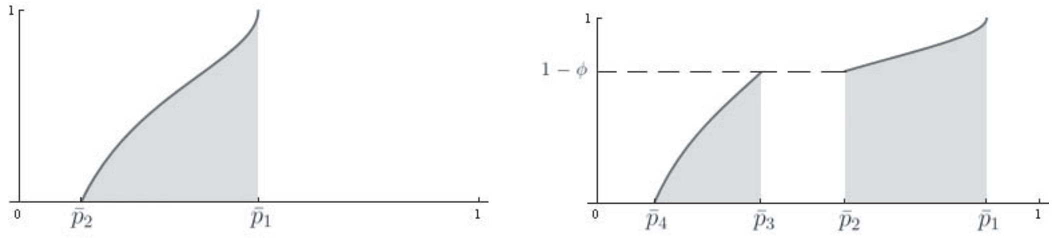
2.4.1 Position-Invariant Costs

In this subsection, we consider the case that the inspection costs are the same for all positions. We let $k_1 = k_2 = k_3 \equiv k$ and $k_1^{'} = k_2^{'} = k_3^{'} \equiv k^{'}$, and we assume $0 < k^{'} < k < 1$. Because we now do not impose any ordering constraint, when there is no difference in the inspection cost for different positions, all positions, and thus all firms, are essentially the same. Naturally, this case reduces to the symmetric random search setting. When non-shoppers have the same search cost, the symmetric equilibrium price distribution can be explicitly derived and analyzed Stahl (1989). When non-shoppers have heterogeneous search costs, like in our setting, the equilibrium analysis is more complicated because non-shoppers may adopt different search strategies, which in turn complicates firms' pricing decisions. Stahl (1996) shows that with continuous cost distribution, a symmetric mixed-strategy equilibrium always exists, although the actual distribution patterns depend on the cost distribution and generally have no closed-form solutions. To better contrast with the local-competition pattern from the previous analysis, we next explicitly derive the symmetric equilibrium under certain conditions.

In the symmetric case, because firms adopt the same pricing strategies in a symmetric equilibrium, the search order becomes trivial: In each step, the rational strategy is either to stop or to randomly pick one of the uninspected firms. In other words, the search strategies simply reduces to the stopping rules. In addition, to determine the stopping rules, the expected gain in Equation (2.6) is easier to calculate. As we can show, when firms adopt the same pricing strategy, the optimal stopping decision based on the expected gain from all the future searches is exactly the same as the decision based simply on the expected gain from the next one search. Therefore, in the symmetric equilibrium where each firm prices according to $F(\cdot)$, consumers' search strategies (with search cost k) can be characterized by an optimal stopping price r^*, which is determined by

$$\int_{\underline{p}}^{r^*} (r^* - p)\, dF(p) - k = 0. \tag{2.7}$$

When the current lowest price exceeds r^*, it is worthwhile to conduct an additional search; otherwise, it is optimal to stop. Thus, in the following symmetric equilibria, we consider consumers' rational search strategies as follows: Shoppers always inspect all positions before making a purchase, and non-shoppers randomly inspect one firm with equal probability. If the price does not exceed their optimal stopping price (denote as r_1^* for type-1 consumers and r_2^* for type-2 consumers), they stop searching and purchase the product. Otherwise, they continue to randomly inspect one of the firms left with equal probability,



(a) $F(p)$ in Example 2.3

(b) $F(p)$ in Example 2.4

Figure 2.3: Examples of Price Distributions in the Symmetric Equilibrium

until the price is no greater than their optimal stopping price or there is no firm left to inspect. They then buy from the one with the lowest price. Therefore, a complete description of the symmetric equilibrium only needs to specify firms' common pricing strategy (represented by the cumulative distribution function F) and non-shoppers' optimal stopping prices (r_1^* and r_2^*).

Proposition 2.2. (Symmetric Pricing with Continuous Support) *There exists a mixed-strategy equilibrium in which all firms price according to the distribution*

$$F(p) = 1 - \sqrt{\frac{\left[1-(1-\alpha)^2\right](\bar{p}_1 - p)}{3(1-\alpha)^2 p}} \quad p \in [\bar{p}_2, \bar{p}_1], \tag{2.8}$$

where $\bar{p}_1 = r_2^*$ *and* $\bar{p}_2 = \frac{1-(1-\alpha)^2}{1+2(1-\alpha)^2} r_2^*$*, and the optimal stopping prices* $r_1^* = r_2^* + k - k'$ *and* r_2^* *is determined by* $\int_{\bar{p}_2}^{\bar{p}_1} F(p)\,dp = k'$*, if the inspection costs* k *and* k' *are commeasurable such that* $0 < r_2^* < r_1^* < 1$ *and* $r_2^* > \frac{1}{2-\alpha} r_1^*$*.*

The next example illustrates the equilibrium described in Proposition 2.2.

Example 2.3. *When* $\alpha = .3$*,* $k = .3$*, and* $k' = .2$*, we can calculate non-shoppers' optimal stopping prices* $r_1^* = .62$ *and* $r_2^* = .52$ *(*$r_2^* > \frac{1}{2-\alpha} r_1^*$*), and firms' price-support bounds* $\bar{p}_1 = .52$ *and* $\bar{p}_2 = .13$*. The price distribution is depicted in Figure 2.3(a).*

When the search cost of the type-1 consumers is not too high relative to that of the type-2 consumers, firms set the upper bound of the price support equal to the optimal stopping price of the type-2 consumer so that all non-shoppers stop searching after sampling once. In this case, the firms forgo the option of charging a higher price to take advantage of the high-cost type-1 consumers because the benefit from exploiting the high-cost consumers cannot counterbalance the loss of business from the low-cost consumers.

Proposition 2.3. (Symmetric Pricing with Segmented Support) *There exists a mixed-strategy equilibrium in which all firms price according to the distribution*

$$F(p) = \begin{cases} 1 - \sqrt{\frac{\alpha\bar{p}_1 - \alpha p - \alpha(1-\alpha)(1+\phi+\phi^2)p}{3(1-\alpha)^2 p}} & p \in [\bar{p}_4, \bar{p}_3] \\ 1 - \sqrt{\frac{\alpha(\bar{p}_1 - p)}{3(1-\alpha)p}} & p \in [\bar{p}_2, \bar{p}_1] \end{cases}, \tag{2.9}$$

where $\bar{p}_1 = r_1^*$, $\bar{p}_2 = \frac{\alpha}{\alpha+3(1-\alpha)\phi^2} r_1^*$, $\bar{p}_3 = r_2^*$, $\bar{p}_4 = \frac{\alpha}{\alpha+3(1-\alpha)^2+\alpha(1-\alpha)(1+\phi+\phi^2)} r_1^*$, *and* $\phi = \frac{-\alpha}{2(3-2\alpha)} + \frac{\sqrt{\alpha(-9\alpha^2+29\alpha-24)+4\alpha(3-2\alpha)r_1^*/r_2^*}}{2\sqrt{1-\alpha}(3-2\alpha)}$, *and the optimal stopping prices* r_1^* *and* r_2^* *are determined by*

$$\begin{cases} \int_{\bar{p}_4}^{\bar{p}_3} F(p)\, dp = k' \\ \int_{\bar{p}_2}^{\bar{p}_1} F(p)\, dp + (\bar{p}_2 - \bar{p}_3)(1-\phi) = k - k', \end{cases} \tag{2.10}$$

if the inspection costs k *and* k' *are different enough such that* $0 < r_2^* < r_1^* < 1$ *and* $\frac{\alpha}{3-2\alpha} r_1^* < r_2^* < \frac{1}{2-\alpha} r_1^*$.

The next example illustrates the equilibrium described in Proposition 2.3.

Example 2.4. *When* $\alpha = .3$, $k = .5$, *and* $k' = .1$, *we can calculate non-shoppers' optimal stopping prices* $r_1^* = .88$ *and* $r_2^* = .37$ ($\frac{\alpha}{3-2\alpha} r_1^* < r_2^* < \frac{1}{2-\alpha} r_1^*$), *and firms' price support* $\bar{p}_1 = .88$, $\bar{p}_2 = .56$, $\bar{p}_3 = .37$ *and* $\bar{p}_4 = .13$. *Also,* $\phi = .29$. *The price distribution is depicted in Figure 2.3(b).*

Intuitively, when the search cost of the type-1 consumers is significantly higher than that of type-2 consumers, the optimal stopping price of the type-1 consumers is thus considerably higher than that of the type-2 consumers. As a result, charging a higher price to exploit the high-cost consumers could be as profitable as charging a lower price to compete for more market share, which explains the rightward expansion of the price support compared to the previous result. Particularly, it is worth noticing that *segmentation* of price supports arises in equilibrium in this case. The gap between $\bar{p}_3$ and $\bar{p}_2$ results from the drop of expected demand for prices right above $\bar{p}_3$. Because $\bar{p}_3$ equals the type-2 consumers' optimal stopping price r_2^*, when pricing below $\bar{p}_3$, a firm can stop all type-2 consumers who inspect its position from further searching. However, once its price exceeds $\bar{p}_3$, the type-2 consumers will continue to inspect other positions and are very likely to purchase from elsewhere (unless all the other positions charge even higher prices). Therefore, the expected demand drops substantially at $\bar{p}_3$. As a result, the expected profit jumps downward at $\bar{p}_3$ and does not rise back until $p \geq \bar{p}_2$. For this reason, charging any price between $\bar{p}_3$ and $\bar{p}_2$ is suboptimal.

One question of our particular interest is: could the local-competition price pattern arise in equilibrium in the case of position-invariant costs? The answer is negative, as we show in Proposition 2.4. In fact, we can conclude that the typical symmetric random search market cannot induce the local-competition equilibrium pricing pattern.

2.4.2 Position-Dependent Costs

We now allow consumers' inspection costs to be different for different positions. Position-dependent inspection costs reflects the inherent difference across different positions, which lies in the difference in terms of visibility and accessibility. Just like bending over to check the bottom-level shelf space in the supermarket could be costly for seniors, scrolling down and looking for an unhighlighted link on a webpage or switching multiple webpages could be troublesome for non-tech-savvy users and/or in the case of unsatisfactory network connection.

We first give a necessary condition for the local-competition pattern to arise in equilibrium.

Proposition 2.4. (Local-Competition Pattern under Endogenous Search: A Necessary Condition) *A similar pricing pattern as in Equation* (2.1) *may arise in a rational-expectations equilibrium only if at least one group of consumers have a higher inspection cost for position 3 than for position 1 (i.e., $k_3 > k_1$ or $k_3' > k_1'$).*

Proposition 2.4 shows that for the pattern to appear in REE, at least some consumers' inspection cost for the third position should be strictly higher than that for the first position. Otherwise, because the price at the first position is always higher than that at the third position, as in Equation (2.1), inspecting the first position would be dominated by inspecting the third one for all consumers. In this case, the rational search decision would be to inspect the second and the third positions only. If this is the case, then both the first and the second firms will deviate from the presumed pricing strategies. As a result, the pattern in Equation (2.1) cannot hold as an equilibrium.

Proposition 2.4 thus excludes the possibility that the local-competition pattern is an equilibrium when there is no inherent difference among different positions. It reveals the fact that the special pattern of equilibrium price dispersion is an outcome of position-dependent inspection costs, which induce certain search ordering of consumers as rational equilibrium behaviors.

We next explicitly derive an equilibrium with the same pricing pattern as Equation (2.1) under certain conditions.

Proposition 2.5. (Local-Competition Pattern under Endogenous Search: A Sufficient Condition) *When non-shoppers' inspection costs for different positions increase with position ranks and the differences are large enough; precisely, when* $k_1 < \frac{\alpha(1-\alpha)-\ln(1+\alpha(1-\alpha))}{1+\alpha(1-\alpha)}\bar{p}_1$, $k_3 - k_2 > \bar{p}_3 \frac{\alpha}{1-\alpha}\ln\alpha + \frac{\bar{p}_1}{1-\alpha}\ln(1+\alpha(1-\alpha))$, $k_1^{'} = 0$, $k_2^{'} < (\gamma^* - \bar{p}_3) - \bar{p}_3 \ln \frac{\gamma^*}{\bar{p}_3}$ *(with* $\gamma^* = \frac{1+2\alpha(1-\alpha)+\sqrt{4\alpha^4-12\alpha^3+12\alpha^2-4\alpha+1}}{2(2-\alpha)[1+\alpha(1-\alpha)]}\bar{p}_1$*), and* $k_3^{'} > \left[1 + \frac{\alpha\ln\alpha}{(1-\alpha)[1+\alpha(1-\alpha)]}\right]\bar{p}_1$, *there exists a rational-expectations equilibrium in which firms price according to*

$$\begin{aligned}
F_1(p) &= \begin{cases} 1 - \frac{\bar{p}_2}{p} & p \in [\bar{p}_2, \bar{p}_1) \\ 1 & p = \bar{p}_1 \end{cases}, \\
F_2(p) &= \begin{cases} 1 - \frac{\bar{p}_3}{p} & p \in [\bar{p}_3, \bar{p}_2) \\ 1 - \frac{(\bar{p}_1 - p)}{(1-\alpha)p} & p \in [\bar{p}_2, \bar{p}_1] \end{cases}, \\
F_3(p) &= 1 - \frac{\alpha(\bar{p}_2 - p)}{(1-\alpha)p} \qquad p \in [\bar{p}_3, \bar{p}_2],
\end{aligned} \tag{2.11}$$

where

$$\bar{p}_1 = \min\{k_2 \left[1 + \frac{\alpha\ln\alpha}{1+\alpha(1-\alpha)} - \frac{\ln(1+\alpha(1-\alpha))}{1-\alpha}\right]^{-1}, 1\},$$

Table 2.1: Consumers' Search Decisions $d(z, \mathcal{C})$

Choice Set $\mathcal{C}$	Type-1 Consumers	Type-2 Consumers
$\{1,2,3\}$	If $z=1$, inspect 1st position	If $z=1$, inspect 1st position
$\{2,3\}$	If $(1 \geq)\, z > \bar{p}_1$, inspect 2nd position; If $z \leq \bar{p}_1$, stop	If $z > r_2$, inspect 2nd position; If $z \leq r_2$, stop
$\{1,3\}$	If $z > r_1$, inspect 1st position; If $z \leq r_1$, stop	If $z > \bar{p}_2$, inspect 1st position
$\{1,2\}$	If $z \leq \bar{p}_2$, stop	If $z > \bar{p}_2$, inspect 1st position
$\{1\}$	If $z \leq r_1$, stop	If $z > \bar{p}_2$, inspect 1st position
$\{2\}$	If $z \leq \bar{p}_1$, stop	If $z > r_2$, inspect 2nd position; If $z \leq r_2$, stop
$\{3\}$	If $z \leq \bar{p}_1$, stop	If $z \leq \bar{p}_1$, stop

1. $\int_{\bar{p}_2}^{r_1} (r_1 - p)\, dF_1(p) = k_1$. (Notice that $\bar{p}_2 < r_1 < \bar{p}_1$.)
2. $\int_{\bar{p}_3}^{r_2} (r_2 - p)\, dF_2(p) = k_2'$. (Notice that $\bar{p}_3 < r_2 < \bar{p}_2$.)

$\bar{p}_2 = \frac{1}{1+\alpha(1-\alpha)}\bar{p}_1$, *and* $\bar{p}_3 = \alpha\bar{p}_2$; *consumers adopt the search strategies specified in Table 2.1.*[4]

The conditions needed are that the inspection costs are higher for the inferior positions than those of the superior ones and that they differ in the same order for all non-shoppers (i.e., $k_1 < k_2 < k_3$ and $k_1' < k_2' < k_3'$). In addition, the differences between the inspection costs for two different positions are heterogeneous across consumers. In other words, while some consumers (i.e., shoppers) are insensitive to location difference, others (i.e., non-shoppers) are quite sensitive, and these non-shoppers have different levels of tolerance toward locational inconvenience. For example, some consumers might not mind scrolling down and looking for an unhighlighted link on the same webpage but would not bother to switch to another webpage to continue their search, or it might be fine for some to bend over and check the bottom shelf in the supermarket but it would be too troublesome to get a ladder and check the top shelf. As an equilibrium outcome of such differences, a commonly observed search order arises in equilibrium, and meanwhile, different consumers stop at different stages of the search process.

[4]Alternatively, we can describe consumers' search strategies based on *Pandora's Rule*, which relies upon a major result from Weitzman (1979). Here, we opt to formulate and solve the whole problem within a self-contained framework.

Anticipating consumers' rational search strategies, firms also optimize their pricing strategies by selectively retaining some consumers from further search, competing for some who sample multiple positions and forgoing the others. As an equilibrium outcome, type-1 consumers choose to start their search from the first position because it yields the highest expected gain according to Equation (2.6). After learning the price from the first position, they decide to stop searching because further inspection of either the second or the third position yields a negative expected gain. Similarly, type-2 consumers stop searching after inspecting the first two positions because of the higher search cost for the third position. Shoppers inspect all three positions before making a purchase, and the order they pursue actually does not matter.

Table 2.1 lists non-shoppers' search decisions $d(z, \mathcal{C})$ when facing values of $\{z, \mathcal{C}\}$ which are relevant to the equilibrium analysis. From the table, we can identify non-shoppers' equilibrium search strategies. For instance, the lowest price is $z = 1$ before consumers start the first search, and the decision facing consumers is $d(1, \{1, 2, 3\})$. For type-1 consumers, according to Table 2.1, $d(1, \{1, 2, 3\})$ is to inspect the first position. After that, they learn a price p from the $F_1(p)$ so that $p \in [\bar{p}_2, \bar{p}_1]$. Then the decision $d(p, \{2, 3\})$ for the type-1 consumers is to stop, which completes the type-1 consumers' search strategies in equilibrium. We also specify consumers' off-equilibrium strategies. For instance, the type-1 consumers will not actually face the decision $d(z, \{1, 3\})$. Nevertheless, this decision and the associated payoff should be taken into account when they calculate the expected gain of inspecting the second position in the first place. Also, the type-2 consumers face the decision $d(z, \{2, 3\})$ after inspecting the first position. Despite the fact that the price quoted from the first position always falls in $[\bar{p}_2, \bar{p}_1]$ in equilibrium, we also need to specify the off-equilibrium search decision when $z < \bar{p}_2$ because such decision affects the first firm's profitability of possible deviation. As we can verify, all the decisions listed in Table 2.1, both in-equilibrium and off-equilibrium, are rational.

2.5 Conclusion

In addition to the theoretical contribution to the literature on search and pricing, this study has managerial implications for sellers, consumers, and information systems designers. For sellers seeking the optimal pricing strategy, we emphasize the importance of recognizing the features of consumers' online search behavior and adjusting the price accordingly. Because the online environment makes it relatively easy to sample around, a competitive price could quickly be noticed and result in a surge of sales. Therefore, it would be beneficial to provide occasional promotions or limited-time deals from time to time, which could not only boost short-term sales but also attract more visits in the long run. Nevertheless, the optimal pricing strategies would avoid being too aggressive or too conservative in price competition. The local-competition pattern suggests competing only against commensurable opponents with similar visibility. In most cases, it would be suboptimal to compete against sellers that are much stronger or weaker. Moreover, to form their optimal pricing strategies, sellers only need "local" information on consumers' search behavior at the positions with similar visibility and on the pricing strategies of the firms at these neighboring positions.

For consumers looking for the best deals, it might sound discouraging that the exact price at a particular position is usually difficult to predict. It could be the case that the firm at a prominent position offers a good deal, or a firm at an inferior position might not charge as low a price as expected. Stopping the search process early is thus rational for consumers who have high search costs. Nevertheless, since the price expectation decreases as the location prominence drops, it is generally rewarding to keep searching, especially for those who are not sensitive to locational inconvenience.

For designers of the online information portals, this study highlights the critical role of the inherent difference among business positions in affecting consumers' inspection costs, which then determine the search behavior and, in turn, sellers' profits. Appropriately differentiating the visibility of links (e.g., by special decoration, large display areas, or color highlighting) is essential to make the superior positions more appealing to sellers. In addition, notice that the advantage of prominent positions would diminish once consumers get fully accustomed to the webpage design and linking structure. In this sense, occasional updating the structural design would also be necessary.

This work can be extended in several directions. The major limitation of the analysis in Section 4 is that it is somewhat incomplete, as we only study the equilibrium with full rationality in the three-firm case under certain conditions. A future extension would be to explore the general case of asymmetric oligopolistic pricing with optimal consumer search. Another interesting direction for future work would be to consider a dynamic setting in which consumers arrive at different times, and firms compete against each other intertemporally. In addition, the local-competition pattern predicted from the current study could be an interesting topic for future empirical investigation of online price dispersion.

Chapter 3

Price Competition and Endogenous Valuation in Search Advertising

3.1 Introduction

Search advertising, in which advertisers bid to be listed alongside search results or content pages for specific keywords, has been recognized as a successful revolution of the traditional online and offline advertising. The Internet advertising market generated \$22.7bn in the United States in 2009 despite the global economic downturn (PwC and IAB, 2010), and the Internet is predicted to surpass newspapers and television to become the leading advertising medium by 2011 (VSS, 2007). Inside this prosperity, search advertising undoubtedly plays a leading role. Search-related advertising accounts for nearly half of the total Internet advertising revenue, outperforms all of its predecessors (e.g., display or email ads), and continues to grow rapidly.

While spending millions of dollars on search advertising, marketing managers always wonder whether a seemingly attractive advertising position is indeed worth pursuing, how much value a premium sponsored slot creates for them, and how to optimally allocate their marketing spending. Should they bid aggressively to win a good position and charge a price premium to compensate for the spending, or should they save money in the bidding and capture demand with price discounts through sales and promotions? From consumers' perspective, one typical question is where to find the best deal. Will the price from a prominent advertising slot be higher or lower than those from the less-prominent positions? This study is intended to answer these questions.

A crucial step leading to the proper evaluation of sponsored advertising slots is to investigate their values *endogenously* within the competitive environment, which would be easily neglected in industrial practice and academic research. Prominent positions (e.g., advertising slots listed on the top of a web page, or highlighted in special color with extra space) usually generate high click-through rates and hence are commonly believed to be highly desirable. The existing literature on position auction mechanism design (e.g., Athey and Ellison, 2010; Liu et al., 2010), which carefully studies how advertisers shade their bids strategically in the bidding competition, usually assumes the value of per-ad-click as exogenously given and fixed. Under such an assumption, a prominent advertising slot that attracts more click-throughs, naturally, creates higher value for advertisers. Nevertheless, whether these clicks lead to final conversion also depends on the pricing of the product. In other words, the value of advertising slots is not realized without considering the price competition outcome. In this study, we suggest that the true value of a particular advertising slot to a particular advertiser should be understood endogenously in the price competition facing that advertiser. We show that, depending on the competitive situation, a more prominent slot may or may not be more valuable, even if it is cost-free. Instead of emphasizing the strategic bidding details, we focus on the value of advertising positions to advertisers and examine advertisers' willingness-to-pay for a prominent slot. We also analyze the resulting price dispersion associated with the bidding outcome. To the best of our knowledge, we are among the earliest to integrate price competition with bidding competition under the search advertising setting.

Like the advertising model itself, the price and bidding competition in search advertising is different by its nature from the traditional pricing and advertising topics in marketing and economics literature. Major distinctions are two: the unique features of online consumer search behavior and the asymmetric nature embedded in the competition for an exclusive advertising resource.

Compared with traditional offline searching, consumers' online search behavior exhibits unique features: (1) a commonly observed search ordering, and (2) a highly diversified consumers' search cost. The first feature originates from the organization of advertisements in search engine result pages. The common format is that the sponsored links are listed in the right column alongside the organic search results, one after another from the top downward. Because of the reading habits and eye-movement pattern of most human beings, consumers usually process the information following the order of the list, from the top downward. Therefore, consumers generally first pay attention to the top advertising slot of the sponsored list, and then the next, and so on, while some of them stop searching before reaching the bottom.[1] The arrangement of advertisements and the resulting ordering of the search creates a huge prominence difference among advertising slots with different ranks. Both online traffic statistics and empirical studies based on clickstream data have shown that the click-through rate attracted by the top link on a web page is generally the highest, and it decreases significantly from the top downward (e.g., Brooks, 2004; Ansari and Mela, 2003; Ghose and Yang, 2009).

The second feature of online search behavior owes to the advance of information technology, which greatly facilitates informational searches. The physical cost to sample a product and get a price quote from a store, which would otherwise be a non-negligible expense with necessary travel to the store, is now only several mouse clicks. In addition, some consumers do derive hedonic utility from shopping online (e.g., Childers et al., 2001): They enjoy the process of searching different places, comparing prices, and finding the best deal, evidenced by those who spend hours surfing the web to shop. Altogether, with the flourishing of the Internet and online search engines, there arises a certain portion of consumers who have a non-positive (zero or even negative) net search cost. We call them *shoppers*. However, not everybody purchasing online has such luxury. The convenience of e-commerce brings many people with stringent time constraints, whose only goal is to find the product with a minimum of time spent. In addition, the information overload with the Internet and the extra skills needed to accomplish computer-based searches add to the cost for some online consumers. Therefore, there also exists a certain number of

[1] Hoque and Lohse (1999) use experimental data showing that, compared with traditional paper media, consumers are more likely to pay more attention to the advertisements near the beginning of the heading in online directories.

consumers who have a positive search cost, whom we call *non-shoppers*.[2]

In addition to the distinctive features of online search behavior, the exclusiveness of the advertising resource and the asymmetry inherent inside the bidding competition distinguish our study from existing ones. Traditional advertising technology allows advertisers to choose their advertising levels independently, which makes the competition outcome less sensitive to the asymmetry among firms' competence. In light of this feature, starting from Butters (1977), the classical economics models of price advertising unanimously consider symmetric competition among advertisers and derive equilibrium outcomes in which all firms choose the same advertising level and adopt symmetric pricing strategy (e.g., Stegeman, 1991; Stahl, 1994). In contrast, in search advertising, the prominent advertising slot is sold via auction and by its nature is exclusive: Only one firm can win a prominent position. Hence, a slight difference in firms' competence would lead to a huge difference between winning and losing the best business location. This type of advertising thus demands that we capture even a small difference in firms' competence and tease out the bidding result explicitly. As asymmetric competition deals with firms that play different strategies in equilibrium, it brings challenging yet intriguing aspects into the analysis, such as the determination of the winning bid and the comparison of the price expectation. This work is among the very few studies that handle asymmetric competition under the price advertising setting.

To study the interaction between pricing and bidding competition under search advertising, we set up a game-theoretic model involving asymmetric competition among firms and capturing the aforementioned features of online consumer search behavior. Firms first compete for advertising slots via auction and then compete for customers in a product market. Firms differ in their competence, which is represented by production cost. Positions are differentiated in their prominence, which reflects the ordering of the list.

[2]Early consumer research shows that a consumer's search effort is determined by various factors, such as time availability, purchase involvement, and attitudes toward shopping, and consumers do not always search thoroughly, even when purchasing expensive items (Beatty and Smith, 1987). A recent empirical investigation shows that online shoppers tend to search very few sites on average (Johnson et al., 2004).

Consumers start searching from the prominent position. The shoppers conduct a thorough search, while non-shoppers stop searching once their reserve price is satisfied. We thus provide an integrated framework to endogenously investigate the value of advertising slots in the context of both bidding and pricing competition, taking into account firms' relative competitive strength as well as consumers' online search behaviors.

Based on this framework, we show that a prominent advertising slot might or might not be desirable. In some cases, even without any extra cost, the prominent position is not desirable for the firm that has a competitive advantage, which underscores our early argument that whether a seemingly prominent position is indeed superior should be determined endogenously, taking competitors' responses in pricing into consideration.The major trade-off facing the firm with a competitive advantage is exploiting the extra demand when winning the prominent position versus charging high prices when letting the weak competitor win the better position. We also find that the bidding outcome might not always be in favor of the firm with a competitive advantage. In some scenarios, the disadvantaged firm is able to outbid its competitor and wins the prominent position.

By analyzing the equilibrium pricing, we identify a unique pattern of two-dimensional price dispersion: First, we show that the presence of shoppers makes any static pricing unstable, and in equilibrium, the *realized* price from the same position varies. Second, because of the common search ordering and the resulting location prominence difference, firms at different positions adopt different pricing strategies, and the *expected* prices from different positions differ. Interestingly, we reveal that the price expectation from a prominent position might not be higher, somewhat contradicting the common wisdom that an expensive location comes with expensive products.

It is worth noting that in addition to online search advertising, we formulate and analyze the whole problem in a way that the model and its implications are also applicable to other settings involving price promotion and location acquisition, from storefront location competition to slotting space allocation in retail stores, as is discussed in more detail in the last section of this chapter. Along this line, there is rich literature on price promotion and geographical location competition. Varian (1980) studies the symmetric mixed-strategy pricing when some "informed" consumers are fully aware of the prices from all firms and the others are "uninformed." Narasimhan (1988) and Raju et al. (1990) consider duopolistic price competition when some consumers are "loyal" to one brand. These works consider only the price competition and take the information structure or the market segmentation as exogenously given. In contrast, we consider both price and bidding competition so that the "guaranteed" demand is acquired endogenously. In addition, the location advantage is exclusive so that which firm wins reflects the subtle interactions embedded in the asymmetric competition, and firms adopt asymmetric pricing strategies upon winning. Other literature focuses on geographical location competition. For example, Dudey (1990) shows that sellers choose to cluster together when buyers incur higher search costs across different locations. In contrast, we explicitly model prominence difference among locations and consider exclusive location choice decisions.

The rest of the chapter is organized as follows. In Section 3.2, we start with a baseline model to capture the essence of our interest, so as to derive neat results and clear insights. We consider two heterogeneous firms competing for a prominent advertising slot to sell products to consumers. We temporarily leave consumers' search behavior exogenous. Section 3.3 details the analysis and derives results from the baseline model. In Section 3.4, we endogenize consumers' search strategies. We first endogenize consumers' choice of search ordering and allow them to deviate from the presumed order. We then endogenize consumers' sequential search decision and let them strategically decide whether to continue or stop searching. We show that the qualitative results derived from the baseline model stay the same. In Section 3.5, we further extend the baseline model along various directions. We show that the main results continue to hold when search advertising is not the only information channel, when consumers have heterogeneous preferences over firms' products, and when there are more than two firms competing. In addition, we provide some supportive observations that are consistent with our modeling results. Section 3.6 discusses the managerial implications and concludes the chapter.

3.2 The Baseline Model

Two firms compete for a prominent advertising position via auction. The winning firm gets placed in the prominent position, which is called *the first position* or *position 1*. The other firm stays at a less prominent position, *the second position* or *position 2*. The prominent advertising position can be interpreted as the top sponsored advertising slot listed on a search engine results page. Depending on whether they win the prominent position, the firms compete for consumers by setting different prices. The firms are selling a homogeneous product. The homogeneity assumption is partially motivated by the high degree of standardization and digitization of products or services on the Internet. More fundamentally, suppressing heterogeneity among products allows us to see clearly how locational effect (and essentially consumers' online search behaviors) alone generates a significant level of price dispersion (for the *same* product). Relaxing this assumption to consider heterogeneous consumer preferences does not change the main insights, as we show later. The firms are differentiated in their competence, which is represented by the marginal production cost. The firm with competitive advantage, termed as *high type* or H, has a lower marginal production cost c_1, while the firm with competitive disadvantage, termed as *low type* or L, has a higher marginal production cost c_2. Without loss of generality, we normalize c_1 to zero and denote c_2 as c, $c > 0$.

There is a continuum of consumers of measure 1. Each consumer has a unit demand of the product. Consumers obtain price information of the product by sampling the advertising slot(s) (e.g., clicking the sponsored link(s)). We assume that all consumers start sampling products from the prominent position. Among them, α ($0 < \alpha < 1$) are *non-shoppers* who sample the first position only. The other $1 - \alpha$ are *shoppers* who sample both positions. Here, we leave consumers' search behavior exogenous to avoid unnecessary technical complexity in the baseline model, while still capturing the characteristics of consumer online search behavior (i.e., a commonly observed search ordering, and highly diversified consumers' search costs). In Section 4, we relax the ordering assumption to allow consumers to choose the search ordering strategically, not necessarily starting from the prominent position. Also, we endogenize consumers' sequential search decisions, showing that the equilibrium outcome coincides with the exogenous assumptions. Again, to keep the baseline model simple and to avoid unnecessary distraction from the demand factor, we assume that all consumers have the same willingness-to-pay for the product, w, $w > c$. Relaxing this assumption to a case with consumers of heterogeneous willingness-to-pay does not change the main results (as is detailed in Appendix C.1). Consumers buy the product when the price is no greater than w. Consumers who sample both positions have perfect recall on price and make the purchase from the firm with the lower price; if the price is the same for both, they randomly pick one, with equal probability.

The determination of the auction outcome is based on a score that equals a unit-price bid times a weighting factor. Each firm submits a per-visit (or per-click) bid b_i ($i \in \{H, L\}$). The weighting factor ω_i equals the expected visits (clicks) if firm i is placed in the first position. In the baseline model, the weighting factors simply equal 1 for both firms. In the extensions, ω_i may take different values depending on different settings. The firm with the highest score $s_i = \omega_i b_i$ wins the first position and pays on a per-visit basis such that the per-visit payment generates a score equal to the second highest score $s_{i'} = \omega_{i'} b_{i'}$; that is, the winning firm i pays $\frac{\omega_{i'}}{\omega_i} b_{i'}$ per click ($\{i, i'\} = \{H, L\}$). The firm that does not win the first position stays in the second one and pays a reserve price, which is assumed to be zero for simplicity.[3]

We consider a two-stage game. In the first stage, the two firms decide their bidding strategies and get placed in the corresponding positions according to the auction rule. In the second stage, the firms price their product, and consumers sample the position(s) and make purchase decisions.[4] Considering the transparency of business information within

[3]Imposing a positive reserve price may involve issues of firms' entry decision; however, it does not change the main results.

[4]The chosen timeline of the game allows us to explicitly examine the values of different advertising positions. It is also a natural timeline applicable to general contexts involving pricing and location acquisition. In fact, the equilibrium outcomes of bidding and pricing competition exhibit similar patterns under different timeline settings. For example, when bidding and pricing decisions are simultaneous or in a reverse order, in equilibrium, the low-type firm is likely to win the first position only if c and α are relatively small, and very similar pricing strategies arise in equilibrium.

the same industry and highly repetitive interaction in search advertising, we follow the common approach in the literature (e.g., Edelman et al., 2007; Varian, 2007) and consider complete information structure (i.e., the game structure, auction rules, and all parameters are common knowledge to both firms).

3.3 Main Results

Along the line of backward induction, we look for a subgame perfect equilibrium. We first consider the second stage price competition given the winning positions and then study the bidding competition in the first stage.

Because of the existence of shoppers, meaning that there exist a certain portion of consumers who search around, know all price information, and purchase from the firm offering a lower price, a slight price cut relative to the competitor can help a firm capture this portion of demand and thus results in a significant increase in the sales profit. As a result, any static prices from the two firms cannot be an equilibrium. In other words, there is no pure-strategy equilibrium in the second stage price competition.

We next explore the mixed-strategy equilibrium. We use $F_i(p)$, $i \in \{H, L\}$, to describe a firm's mixed strategy of pricing. Like regular cumulative distribution functions, $F_i(p)$ measures the probability that the firm will charge a price less than or equal to p.

Lemma 3.1. *(i) Given that H wins the first position, the equilibrium mixed strategies of pricing in the second stage are*

$$\begin{aligned} F_H(p) &= \begin{cases} \frac{p-m}{p-c} & p \in [m, w) \\ 1 & p = w \end{cases} \\ F_L(p) &= \begin{cases} \frac{p-m}{(1-\alpha)p} & p \in [m, w) \\ 1 & p = w \end{cases} \end{aligned} \tag{3.1}$$

where $m = \max\{\alpha w, c\}$*;*[5]

[5] To be rigorous, when $m = c$, we define the value of $F_H(p)$ at c as $\lim_{p \to c^+} \frac{p-c}{p-c} = 1$.

(ii) Given that L wins the first position, the equilibrium pricing strategies are

$$
\begin{array}{ll}
F_H(p) = \frac{p-c-\alpha(w-c)}{(1-\alpha)(p-c)} & p \in [c+\alpha(w-c), w] \\
F_L(p) = \begin{cases} \frac{p-c-\alpha(w-c)}{p} \\ 1 \end{cases} & \begin{array}{l} p \in [c+\alpha(w-c), w) \\ p = w \end{array}
\end{array}
\tag{3.2}
$$

The above lemma describes an asymmetric mixed-strategy equilibrium in the second stage price competition under different situations. Notice that when H wins the first position and cost advantage is the dominating factor (specifically, $c \geq \alpha w$), H would play pure strategy and charge a competitive price equal to L's marginal cost, so as to grab the entire market; Meanwhile, L mixes over prices, ensuring that H has no profitable deviation, and earns zero profit itself. In this sense, this scenario is close to a standard Bertrand competition with asymmetric production costs.[6]

In all other cases, both firms play mixed strategies and achieve positive expected profit. Similar to other asymmetric mixed-strategy equilibria found under different settings (e.g., Amaldoss and Jain, 2002), the equilibrium pricing strategies should satisfy the following properties. First, both firms' equilibrium price distributions have a common and continuous support such that $F_H(p)$ and $F_L(p)$ are strictly increasing on the common support $[\underline{p}, \bar{p}]$. Otherwise, pricing within those non-overlapped ranges would lead to a sub-optimal profit level. Second, there is no mass point in both firms' distributions on $[\underline{p}, \bar{p})$ such that F_H and F_L are continuous on $[\underline{p}, \bar{p})$. This is because a mass point in one firm's price distribution would result in a downward jump of the other firm's expected demand at that point and consequently lower profit levels in a contiguous region on the right side of that point. In addition, by similar arguments, at most one firm may have a mass point at $\bar{p}$. These properties ensure that the equilibrium identified in the above lemma is a unique equilibrium in the second stage pricing game.

Given that H wins the first position and $\alpha w > c$, the highest price that H can charge is consumers' willingness-to-pay w, which ensures H a profit level of αw by exploiting all

[6]When there is a finite minimum money increment ε, L pricing c and H pricing $c - \varepsilon$ can be a pure-strategy equilibrium in this scenario. Here, we follow the convention and treat money as infinitely divisible. As we can see, either case does not affect the main results.

the surplus of its guaranteed demand α. In light of this line, H will never take any price below αw, since charging a lower price would certainly lead to a profit lower than αw. This explains the price range $[\alpha w, w]$. Similarly, in the case where L wins the first position, L can earn at least $\alpha(w-c)$ by charging w such that it will never take any price lower than $c+\alpha(w-c)$.

Notice that when adopting mixed strategies, firms earn the same expected profit across the price range involved. Therefore, firms' expected profits can be specified by examining the expected profits when firms charge the lowest equilibrium prices, as is summarized in Table 3.1, in which π_i^j denotes firm i's expected sales profit in position j.

Table 3.1: Firms' Expected Profits in Different Situations

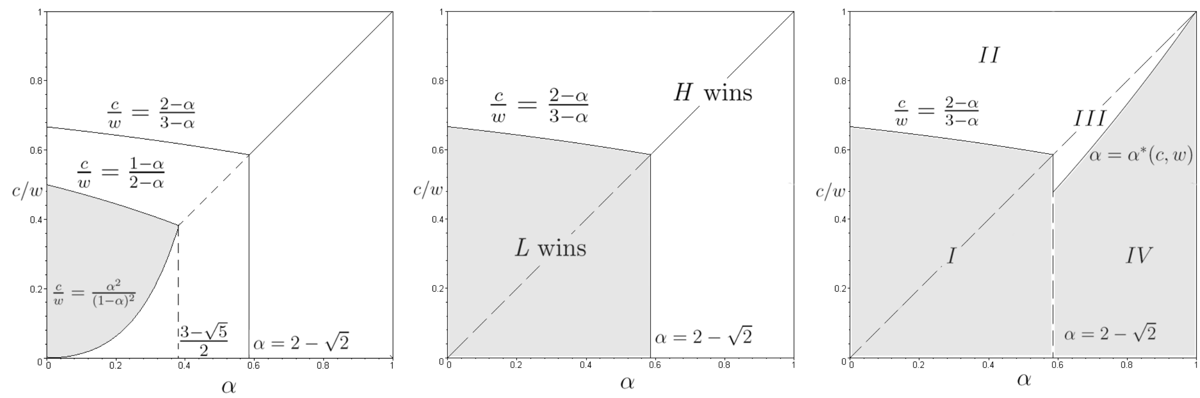
	When H wins	When L wins
H's expected profit	$\pi_H^1 = m$	$\pi_H^2 = (1-\alpha)(c+\alpha(w-c))$
L's expected profit	$\pi_L^2 = (1-\alpha)(m-c)$	$\pi_L^1 = \alpha(w-c)$

Note: $m = \max\{\alpha w, c\}$

One question of particular interest is whether a prominent position is always desirable, which means whether a firm can achieve higher expected profit in the prominent position than otherwise. The following proposition shows that a seemingly prominent position is not always desirable.

Proposition 3.1. (Endogenous Valuation) *For the low-type firm, staying in the first position always brings higher profit than staying in the second one; for the high-type firm, however, when $\frac{\alpha^2}{(1-\alpha)^2}w < c < \frac{1-\alpha}{2-\alpha}w$, the second position is more profitable.*

As is illustrated in Figure 3.1(a), in the shadowed region, H can achieve higher expected profit in the less prominent position than in the prominent one. This surprising result reveals the nature of price competition and captures the trade-off facing the high-type firm between exploiting the prominent location and benefiting from its lower cost. Intuitively, once winning the prominent position, the low-type firm exploits it thoroughly by charging a relatively high price, since it is at a cost disadvantage and the prominent position ensures a guaranteed demand. For this reason, even though staying in the second position and receiving less attention, the high-type firm may earn reasonable revenue by grabbing most of the residual demand at a relatively high price. On the other hand, when it is at the prominent position, the high-type firm may face fierce competition from the low-type firm on the residual demand, since the low-type firm is desperate to appeal to some consumers. Therefore, when the the loss of demand on the less prominent position is not too high (i.e., α is small), staying in the second position could be more profitable for the high-type firm as long as the benefit from the cost difference is not too low (i.e., $c > \frac{\alpha^2}{(1-\alpha)^2}w$). Nevertheless, the cost advantage cannot be too high either; otherwise, the high-type firm could enjoy the profit from the prominent position by excluding its opponent and occupying the entire market, which far exceeds what it can earn in a less prominent position. This explains $c < \frac{1-\alpha}{2-\alpha}w$. In brief, whether the prominent position is worth pursuing for the firm with competitive advantage essentially depends on the trade-off between capturing the non-shoppers (when winning the advantageous position) and charging higher price premium to the shoppers (when letting the weaker opponent win).



(a) Endogenous Valuation (b) Bidding Outcome (c) Price Dispersion

Figure 3.1: Equilibrium Outcomes: The Baseline Model

Proposition 3.1 illustrates a scenario where a prominent advertising position, although generating more clicks, may have less value than a less prominent one for the high-type firm, which indicates that the per-click value of the prominent position could be significantly different from that of the less prominent one. The result underscores that rather than blindly pursuing a prominent advertising position with a high click-through rate, firms should evaluate an advertising position endogenously within the actual competitive environment, taking into consideration firms' relative competence and consumers' search pattern.

Meanwhile, in many cases, the prominent position *does* have its value, and thus both firms will compete for the position. We next consider the bidding competition in the first stage. We first derive firms' equilibrium bidding in a general way to show clearly to what degree the equilibrium bidding depends on the model parameters. Recall that the firm with the higher score $s_i = \omega_i b_i$ wins, where b_i is the per-click bid and ω_i is the weighting factor, and $i \in \{H, L\}$. Also, denote the expected clicks when firm i stays in the first position as λ_i. Therefore, if $\omega_i b_i > \omega_{i'} b_{i'}$, firm i wins the first position, pays $\frac{\omega_{i'}}{\omega_i} b_{i'}$ per click, and achieves a net expected profit level equal to $\pi_i^1 - \lambda_i \frac{\omega_{i'}}{\omega_i} b_{i'}$; otherwise, firm i stays at the second position with net expected profit π_i^2. Suppose $\pi_i^1 > \pi_i^2$; then it is profitable for firm i to outbid its rival (i.e., to bid $b_i > \frac{\omega_{i'}}{\omega_i} b_{i'}$) if and only if $\lambda_i \frac{\omega_{i'}}{\omega_i} b_{i'} < \pi_i^1 - \pi_i^2$. Therefore, bidding $b_i = \frac{\pi_i^1 - \pi_i^2}{\lambda_i}$ is firm i's weakly dominant strategy. Thus, we show that in equilibrium, independent of the weighting factors ω_i, firms bid in such a way that the total willingness-to-pay $(b_i \lambda_i)$ equals $\pi_i^1 - \pi_i^2$. Moreover, since we let the weighting factor ω_i equal the expected clicks λ_i, which is commonly believed to be the major search engines' practices, the score $s_i = \omega_i \frac{\pi_i^1 - \pi_i^2}{\lambda_i} = \pi_i^1 - \pi_i^2$, and thus advertisers are ranked essentially according to their endogenous valuation of the prominent position. Here, in the baseline model, the situation is simpler because $\omega_i = \lambda_i = 1$ for $i = H, L$. As a result, $b_i = \max\{\pi_i^1 - \pi_i^2, 0\}$, and the firm with a higher bid wins the position. By comparing b_H and b_L under different situations, we can conclude the equilibrium bidding outcome, as is summarized in Proposition 3.2 and illustrated in Figure 3.1(b).

Proposition 3.2. (Bidding Outcome) *(i) When $\alpha < 2 - \sqrt{2}$ and $c < \frac{2-\alpha}{3-\alpha} w$, the low-type firm wins the first position; (ii) When $\alpha > 2 - \sqrt{2}$ or $c > \frac{2-\alpha}{3-\alpha} w$, the high-type firm outbids its rival.*

Proposition 3.2 reveals an asymmetric equilibrium in the bidding for the prominent position. It provides the rationale for firms to determine their spending on location competition based on their relative competitive strength and consumers' search behavior, so as to better position themselves in the marketing campaign. This proposition implies that when either the competence difference or the prominence difference is a dominating factor, the firm with a competitive advantage should compete aggressively to acquire the prominent position.

The intuition is as follows. A firm with a cost advantage can easily outperform its competitor in the price competition and grab most of the market share. Therefore, a prominent location is worth pursuing only if a significant difference in prominence exists; otherwise, as long as a certain portion of consumers will visit both sites, the firm can stay in a less prominent position, still win over most of the residual demand, and attain a satisfactory profit level. This explains why only a high-α position motivates the high-type firm to bid aggressively. On the other hand, a firm with a cost disadvantage suffers a lot when its cost disadvantage is huge, which greatly diminishes the profitability of staying in a prominent position. Therefore, the prominent position is more desirable to the low-type firm when c is relatively small.

An interesting aspect revealed by this proposition is that the bidding outcome may not always be in favor of the firm that has a competitive advantage. When endogenously considered within the integrated framework, firms' relative competitive strength may not align with the competition result. In some scenarios, although the high-type firm *does* value the prominent position and wishes to win, it cannot afford to bid as high as the low-type firm. This result is in contrast to the efficient allocation property of the auctions (i.e., high-type bidder wins the prominent position) revealed in the framework with exogenous bidder valuations (e.g., Liu et al., 2010).

It is worth mentioning that the above equilibrium outcome incorporates the extreme cases, namely, $c = 0$, $\alpha = 0$, or both. When the cost difference disappears ($c = 0$), meaning that firms are homogeneous, the model reduces to the commonly seen symmetric competition model, in which symmetric bidding and pricing strategies arise in equilibrium. By Table 3.1, $\pi_H^1 - \pi_H^2$ and $\pi_L^1 - \pi_L^2$ will become the same. Therefore, they will bid equally at $b = \alpha^2 w$ and achieve the same net profit $(1-\alpha)\alpha w$, regardless of whether they win the first position or not. Moreover, Eq.(3.1) and Eq.(3.2) will give the same equilibrium pricing strategy of H and L when in the same position. Likewise, when the location prominence difference vanishes ($\alpha = 0$), meaning that all consumers visit both positions, it reduces to the typical Bertrand competition. Since the two positions thus become the same, neither firm is willing to spend any money in the bidding. By Table 3.1, $\pi_H^1 = \pi_H^2 = c$ and $\pi_L^1 = \pi_L^2 = 0$. Regardless of their positions, H always charges $p_H = c$ whereas L plays the same mixed strategy $F_L(p) = \frac{p-c}{p}$, according to Eq.(3.1) and Eq.(3.2). If both $\alpha = 0$ and $c = 0$, then we arrive at the trivial case where both firms charge the competitive price $p = 0$ and gain zero profit.

Next we examine the prices from different positions. Intuitively, since bidding is costly, the winner of the prominent position tends to charge a higher price to compensate itself, and hence the expected price from the prominent position is supposed to be higher. Nevertheless, as is summarized by the following proposition, it might not always be the case.

Proposition 3.3. (Price Dispersion) *(i) When $\alpha < 2-\sqrt{2}$ and $c < \frac{2-\alpha}{3-\alpha}w$, the expected price from the first position is higher than that from the second one. (ii) When $\alpha > 2-\sqrt{2}$ or $c > \frac{2-\alpha}{3-\alpha}w$, if $\alpha > \alpha^*(c,w)$, the expected price from the first position is higher; if $\alpha < \alpha^*(c,w)$, the opposite is true, where $\alpha^*(c,w)$ is determined by*

$$(\alpha^* - \frac{c}{w}) \ln \frac{1 - \frac{c}{w}}{\alpha^* - \frac{c}{w}} + \alpha^* + \frac{\alpha^*}{1-\alpha^*} \ln \alpha^* = 0 \tag{3.3}$$

Interestingly, the price expectation from the prominent position may be lower when the high-type firm has a significant cost advantage that is not overwhelmed by the prominence difference. In this case, the high-type firm wins the prominent position and would rather take advantage of its low cost to grab more market share via intense price cutting. As is illustrated by the unshadowed region (II and III) in Figure 3.1(c), region II accounts for the case when the high-type firm plays pure strategy, while region III accounts for the mixed-strategy case, in which the high-type firm puts the most mass on the price range close to the lower bound of the support, yielding a lower price expectation.

In all other cases, as can be expected, the prominent position winner reaps its location advantage by charging non-shoppers a higher price in general (shadowed region in Figure 3.1(c)). In fact, when the low-type firm wins the first position (region I in Figure 3.1(c)), according to Eq.(3.2), its price distribution first order stochastically dominates that of the high-type firm.

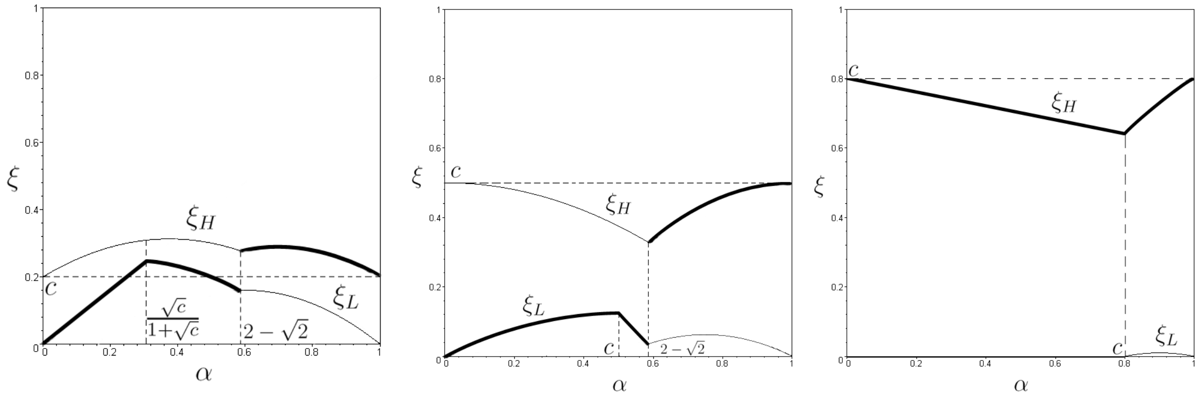
Notice that we derive two dimensions of price dispersion at the same time from the model. First, firms mix their prices in equilibrium, indicating that rather than charging one price for sure, they price with uncertainty. Therefore, the *realized* price from the same position can vary over a certain range probabilistically. This feature coincides with the complexity and uncertainty in determining the final prices of products observed in reality (e.g., different shipping and handling fees, various coupon discounts and cash rebates, etc.). Moreover, as is proposed by Varian (1980), when considered over a long time period, the mixed-strategy pricing can lead to price fluctuation over time (e.g., with occasional promotions or mark-ups), which accounts for the *temporal* price dispersion. Second, firms at different positions adopt different pricing strategies in equilibrium so that the *expected* prices from different positions differ, which accounts for the *spatial* price dispersion. Empirical evidences of online price dispersion in both dimensions have been well documented in the literature.[7]

It is worthwhile to pinpoint the driving forces of the unique two-dimensional price dispersion pattern derived here. The first dimension is due to the presence of shoppers who always search around and thus make any static pricing unstable. The spatial dispersion

[7]Smith and Brynjolfsson (2001) and Chen and Hitt (2002) show that significant levels of price dispersion exist online across different firms, even after control for various heterogeneities. Baye et al. (2004) find that the identities of the lowest-priced firms for various online products keep changing over time, which suggests a persistent level of temporal price dispersion. See Pan et al. (2004) for a detailed literature review.

originates from the search ordering and its resulting prominence difference. With the majority of consumers observing a common search ordering and the existence of non-shoppers, a prominent position gets its prominence advantage by easily attracting consumers' attention and retaining a portion of them. Such an asymmetric prominence leads to different expected prices for different positions. Moreover, the two dimensional dispersion is further enriched by the asymmetry in advertisers' competitive strength. Partially reflecting the bidding outcome, spatial dispersion can occur in one way or the other, depending on the competence difference (compared to the prominence difference).

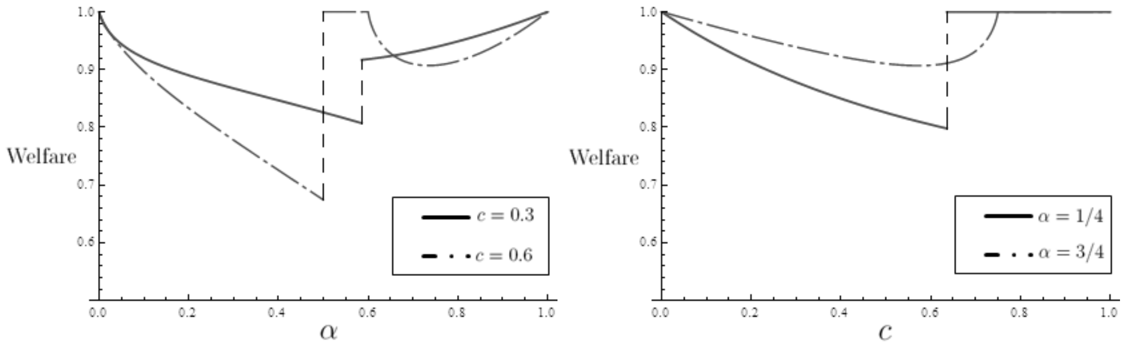
Next, we present some comparative statics results on how model parameters affect advertisers' net profits (i.e., sales profit net of bidding cost), social welfare, and the revenue of the advertising provider (e.g., the search engines) in equilibrium.



(a) $w = 1$, $c = 0.2$ (b) $w = 1$, $c = 0.5$ (c) $w = 1$, $c = 0.8$

Figure 3.2: The Changes of Advertisers' Net Profits (ξ) in α

For advertisers' net profits, simple algebra shows that in equilibrium, the high-type firm always achieves higher net profit than the low-type firm, despite all the complexity of gain and loss in the bidding and price competition. This result simply implies that the cost advantage is by all means rewarding. Furthermore, Figure 3.2 shows how both firms' net profits change with α given different c, where the bold curves highlight the net profit of the prominent position winner. Notice that neither firm's equilibrium net profit changes monotonically in α. The non-monotonicity originates from two counteracting effects that α impacts the sales profit at the second position: On the one hand, a larger α takes away more market share. On the other hand, a larger α leaves higher profit margins, since the prominent position winner tends to charge a higher price. Since the winner has to pay a total price equal to the competitor's profit difference between staying in the two positions, such non-monotonicity is thus introduced into the bidding cost and then into the net profit. An interesting aspect of this result is that even from the prominent position winner's perspective, a higher prominence advantage α may not necessarily lead to a higher equilibrium net profit.



(a) The Changes of Social Welfare in α ($w = 1$) (b) The Changes of Social Welfare in c ($w = 1$)

Figure 3.3: The Changes of Social Welfare in α and c

The overall social welfare equals the sum of consumers' surplus, advertisers' net profit, and the advertising provider's revenue. Essentially, it equals the total consumer value realized from the consumption of the products minus firms' production costs. For example, when L wins the first position in equilibrium, the expected social welfare can be written as:

$$W = w - \left[\alpha + (1-\alpha) \int_{\underline{p}}^{\bar{p}} F_L(p)\, dF_H(p)\right] c, \tag{3.4}$$

where the integral represents the probability of L's charging a lower price than H, which in turn measures L's expected sales to the shoppers. Figure 3.3 illustrates how the welfare changes in α and c under different scenarios. Notice that the jumps depicted in both (a) and (b) correspond to the points when the bidding outcomes reverse (H wins on the right-hand side of the jumps and L wins on the left). As we can see, the "V"-shaped welfare curves indicate that once the winning (low-type) firm causes too much welfare loss, it automatically gets replaced by its competitor. In this sense, in allocating the exclusive advertising resource, the auction mechanism serves as an auto-adjustment to prevent substantial welfare loss.

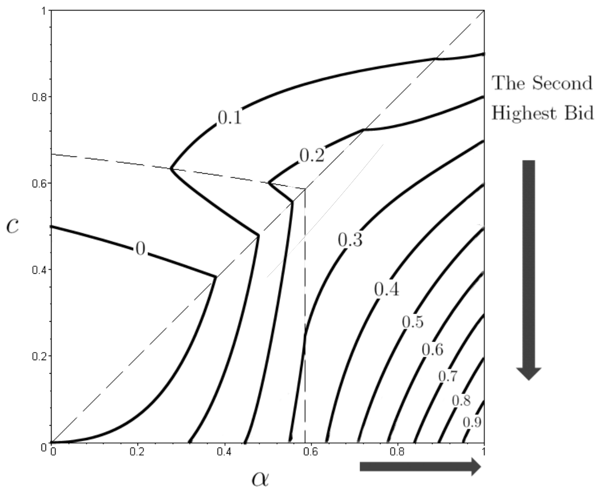


Figure 3.4: The Level Curves of Search Engine's Revenue

The search engine's revenue is determined by the winning firm's payment, which in turn is determined by the second highest bid. Thus, it also reflects the competitiveness of the bidding. Figure 3.4 depicts the level curves of the search engine's revenue. As we can see, the revenue increases rapidly toward the right bottom corner. Small c means that the two firms are close to each other in terms of competitive strength. Meanwhile, a large α indicates a huge difference between winning the prominent position and not. The combination of the two induces firms to compete ultra-intensively for the prominent position.

3.4 Endogenous Consumer Search

In the baseline model, we leave consumers' search behaviors as exogenous to induce easily understood analysis, neat results and clear insights, while avoiding too much technical complexity. In this section, we endogenize consumer search. In particular, we consider consumers' strategic choice of search ordering by allowing them to start searching from the position with a lower expected price; we also consider consumers' endogenous sequential search decision (i.e., whether to conduct another search) by allowing them to rationally assess the expected gain from an additional search. As we show, the qualitative results from the baseline model remain the same.

3.4.1 Strategic Choice of Ordering

Under the assumed consumer search ordering, we have shown that spatial price dispersion does exist, and the expected price from the prominent position could be higher. Understanding this, some sophisticated consumers may anticipate firms' pricing strategies, and start sampling from the position with the lower (expected) price instead of simply following the presumed search behavior. We now consider the case in which some consumers are *sophisticated* and strategically choose their search ordering. We continue to consider the diversification among consumers' search behavior—shoppers and non-shoppers coexist. Notice that for sophisticated shoppers, it actually does not matter which position they start with. For the sophisticated non-shoppers, however, their rational behaviors may affect firms' decision and alter the competitive picture to some extent.

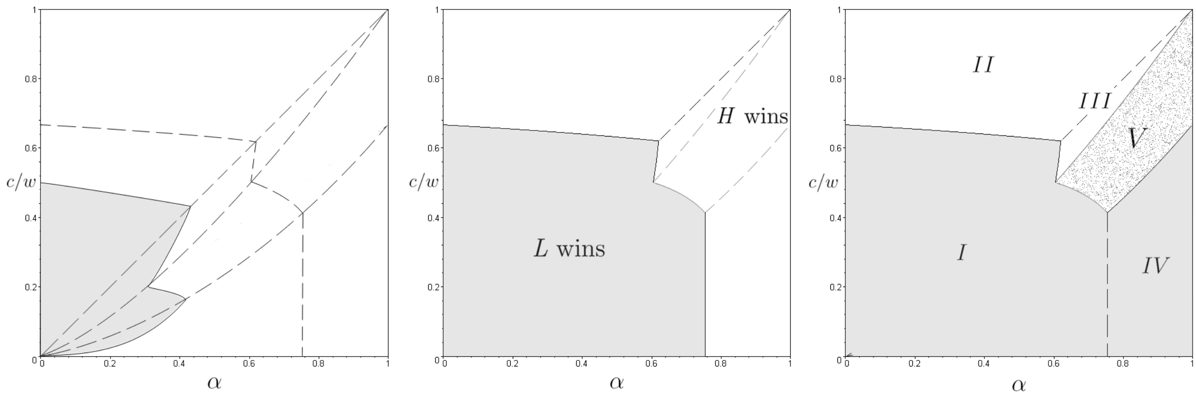
Following the framework of the baseline model, we continue to assume that α of the consumers are non-shoppers who sample only once while $1-\alpha$ are shoppers who sample both positions. Now we consider that among all the consumers, a small portion of them, β ($0 < \beta < 1$), are *sophisticated*, and they can anticipate firms' strategies and start sampling from the position with the lower (expected) price. The rest of them ($1-\beta$) simply start sampling from the first position. In other words, $1-\alpha$ of all the consumers sample both positions (it does not matter which position they start with), while $\alpha\beta$ of the consumers sample the position with the lower (expected) price only and the rest $\alpha(1-\beta)$ only look at the first position. In the first stage, firms decide their bidding strategies and obtain different positions according to the auction rules. In the second stage, sophisticated consumers observe the bidding outcome and decide which position to start with, and meanwhile firms price their product. Then, all consumers sample the position(s) and make purchase decisions.

The strategy profile can be written as $\{b_i, F_i(\cdot; s_H, s_L), \sigma(s_H, s_L) : i \in \{H, L\}\}$, where, as in the baseline model, b_i is firm i's per-click bid in the first stage and $F_i(\cdot; s_H, s_L)$ is the cumulative distribution function of firm i's pricing strategy in the second stage, contingent on the bidding outcome (i.e., the comparison of the bidding scores s_H and s_L). Here we use $\sigma(s_H, s_L)$ to describe the strategy of those sophisticated consumers: Observing the outcome of the auction, they start sampling from the first position with probability σ , and they start from the second position with probability $1 - \sigma$, $0 \leq \sigma \leq 1$. For a strategy profile to be a *subgame-perfect rational-expectations* equilibrium, it should satisfy the following two conditions: First, given the outcome of the bidding competition in the first stage, $\{F_i(\cdot; s_H, s_L), \sigma(s_H, s_L) : i \in \{H, L\}\}$ must be a rational-expectations equilibrium. Specifically, given the assigned positions and sophisticated consumers' strategy, the firms have no profitable deviation in their pricing strategies in the second stage. Meanwhile, sophisticated consumers are rational, which means that their belief about which position has a lower expected price is consistent with firms' equilibrium outcome. That is, $\sigma = 1$ if $E(p^1) < E(p^2)$; $\sigma = 0$ if $E(p^1) > E(p^2)$; $0 < \sigma < 1$ only if $E(p^1) = E(p^2)$, where $E(p^j)$ is the expected price from the position j, $j \in \{1, 2\}$. Second, anticipating the equilibrium play in the second stage, the firms have no profitable deviation in their bidding strategies in the first stage.

As we can see, now the price competition consists of two levels: One is to compete for sophisticated non-shoppers by price expectation, and the other is to compete for shoppers by realized price.

We can conduct a similar analysis as in the baseline model, though with more complexity. (A brief analysis can be found in Appendix C.2.) Figure 3.5 illustrates the equilibrium outcome with $\beta = \frac{1}{4}$. Following the basic patterns found in the baseline model, in the shadowed region in Figure 3.5(a), the high-type firm does not value the prominent position at all. The high-type firm can win the auction only when either prominence difference or competence difference is significant (the unshadowed region in Figure 3.5(b)), while the low-type firm outbids its rival when both α and c are small (the shadowed region).

The expected price from the prominent position is lower when a significant competence difference is not overwhelmed by the prominence difference (the unshadowed region in Figure 3.5(c)). If the cost difference is not so salient, however, the expected price from the prominent position will be higher (the shadowed region).



(a) Endogenous Valuation (b) Bidding Outcome (c) Price Dispersion

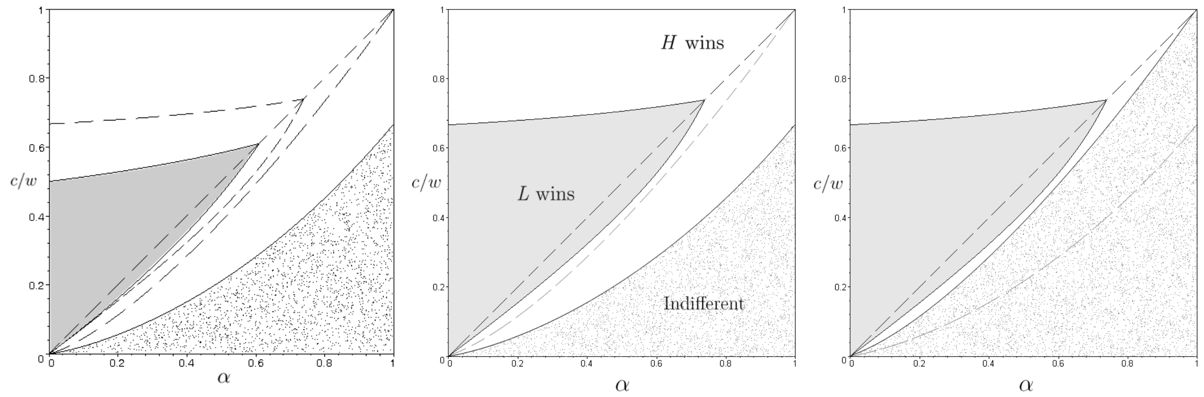
Figure 3.5: Strategic Choice of Ordering: $\beta = \frac{1}{4}$

Further scrutiny may reveal the effects of consumers' strategic ordering choice on the equilibrium outcomes. Comparing Figure 3.5(b) with Figure 3.1(b), the shaded region expands as a result of the presence of the sophisticated consumers, which implies that the low-type firm has a greater chance of winning the prominent position. This is certainly not because the low-type firm becomes more competitive here. Instead, it means that the prominent position becomes less attractive to the high-type firm as the portion of sophisticated consumers increases. By staying at the less prominent position and charging a lower price in expectation, the high-type firm not only has a greater chance to win over shoppers but also can attract sophisticated non-shoppers. Expecting the extra demand from the sophisticated consumers, compared to the case without sophisticated consumers, the high-type firm feels less motivated to acquire the prominent position. This trend is clearly evidenced by the expansion of the "no-interest" region in Figure 3.5(a) (compared to that region in Figure 3.1(a)).

Regarding the expected prices, in the unshadowed region of Figure 3.5(c), the high-type firm charges a lower expected price and thus attracts all the sophisticated consumers.

In the shadowed region, the winning firm charges a higher expected price and thus forgoes those sophisticated consumers to its competitor. The winning firm does not charge a lower expected price to attract the sophisticated consumers either because the winning firm is the low-type firm, which has a cost disadvantage, or because the prominence advantage is salient. The dotted region, in which the expected prices from the two positions are the same and thus sophisticated consumers are indifferent in sampling the first or second position, serves as a transition between the two deterministic cases. In this region, the high-type firm wins the prominent position, and the relatively intermediate advantage of the location prominence compared with its cost advantage makes any deterministic choice by sophisticated consumers unsustainable. On the one hand, if all sophisticated consumers start from the first position for sure, the guaranteed demand becomes too significant to prevent H from exploiting them with a higher price, which contradicts sophisticated consumers' expectation. On the other hand, the relative prominence advantage is not salient enough such that H can afford to forgo all those sophisticated consumers. As a result, H charges the same expected price as L and grabs some of the sophisticated consumers, and sophisticated consumers randomize their sampling strategies. With the emergence of such a middle region, in which the expected prices from the two positions are the same, we can see that consumers' strategic ordering choice reduces the price dispersion. As the portion of sophisticated consumers increases, firms are more likely to charge the same expected price level.

As the portion of sophisticated consumers further grows, the equilibrium pattern slightly changes while the main results of interest remain. Figure 3.6 illustrates the equilibrium outcomes when $\beta = \frac{3}{4}$. One distinctive feature is the existence of a region in which both firms are indifferent about winning the prominent position (the dotted region in Figures 3.6(a) and 3.6(b)). The reason is as follows. Since the portion of sophisticated consumers is significant enough that their deterministic choice could create a large amount of guaranteed demand, neither type of firm can resist charging a higher price for this guaranteed demand, which in turn contradicts sophisticated consumers' rational expectations.



(a) Endogenous Valuation (b) Bidding Outcome (c) Price Dispersion

Figure 3.6: Strategic Choice of Ordering: $\beta = \frac{3}{4}$

Therefore, no matter which firm wins the prominent position, sophisticated consumers playing pure strategy cannot arise as an equilibrium. Instead, they randomize their sampling in equilibrium. It is such mixed strategy that gives either firm the same share of guaranteed demand from being in the two positions, which results in firms' indifference between them. In consequence, the price dispersion is further reduced and the expected prices from the two positions are the same in a large region (dotted region in Figure 3.6(c)). When all consumers are sophisticated (i.e., $\beta = 1$), it reduces to an extreme case in which the first position's "advantage" vanishes completely and there is essentially no difference between the two positions.

3.4.2 Endogenous Sequential Search

The analysis so far treats the consumer sequential search decision as exogenous, that is, some portion of consumers are assumed to search only once, while others are assumed to sample both positions. This seemingly strong assumption, in fact, reflects the equilibrium outcome when consumers are allowed to make their sequential search decision endogenously. We now extend the baseline model to endogenize consumers' sequential search decision (i.e., whether to continue or stop searching). As we shall show, as long as both a commonly observed search ordering and a certain portion of consumers with non-positive search cost exist, the equilibrium bidding outcome and price dispersion pattern derived from the baseline model continue to hold.

Following the framework in the baseline model, we modify the setup about consumer search behavior as follows. Section 3.4.1 has explained the effect of strategic choice of ordering; Here, we assume that all consumers follow the presumed search ordering to simplify analysis. Suppose that all consumers first sample the prominent position and learn the price, and then they assess the expected gain from an additional search. If the expected gain from the additional search exceeds the search cost, they proceed to sample the second position, compare the prices, and purchase from the position with the lower price; otherwise, they stop searching and purchase from the first position (provided the price does not exceed their willingness-to-pay w). Following the common assumption in literature (e.g., Stahl, 1989), we assume that all consumers sample at least one position. We consider consumers with heterogeneous search costs. Particularly, assume that $1-\alpha$ of the consumers have zero search cost, and α of them have positive search cost k $(0 < k < w)$.[8] Similar as before, firms first submit their bids, then firms and consumers observe the bidding outcome and decide pricing and searching strategies simultaneously.

We first characterize consumers' optimal searching strategies. Given the pricing strategy of the firm in the second position $F(\cdot)$ (which again is a cumulative distribution function defined on a price support $[\underline{p}, \bar{p}]$), for any individual consumer, the expected gain from sampling the second position, after already knowing the price p from the first position, can be formulated as follows:

$$G(p) = \int_{\underline{p}}^{p} (p - x) dF(x) \tag{3.5}$$

Notice that the price from the first position, p, must belong to the interval $[\underline{p}, \bar{p}]$ due to the common support properties discussed earlier. As $G(p) \geq 0$, it is always optimal for those shoppers with non-positive search cost to conduct an additional search. For those non-shoppers with positive search cost, however, it is worthwhile to sample the second position

[8] Alternatively, we might allow k to vary following a certain distribution over some positive interval. In that case, the mixed-strategy pricing continues to be the only possible outcome, but there is generally no closed-form solution (similar to Stahl, 1996).

only if $G(p) > k$. Similar to Weitzman (1979), we can equivalently define a reserve price r, such that

$$\int_{\underline{p}}^{r} (r - x) dF(x) = k \tag{3.6}$$

When the price quoted from the first position, p, exceeds r, it is profitable for non-shoppers to conduct an additional search; otherwise, they will stop searching and purchase from the first position.

After characterizing consumers' optimal searching strategies, we can specify the equilibrium concept. A subgame perfect equilibrium is a strategy profile $\{b_i, F_i(\cdot; s_H, s_L), r(s_H, s_L) : i \in \{H, L\}\}$ such that: First, observing the bidding outcome, $\{F_H(\cdot), F_L(\cdot), r\}$ is an equilibrium in the second stage. In other words, given both firms' pricing strategies $F_H(\cdot)$ and $F_L(\cdot)$, shoppers always sample both positions, while non-shoppers sample the first position and learn the price p, and proceed to sample the second position if and only if $p > r$, where r is defined by Eq.(3.6) by substituting the pricing strategy of the firm at the second position for $F(\cdot)$. Meanwhile, given consumers' sequential search strategy (specified by r) and the other firm's pricing strategy $F_{i'}(\cdot)$, either firm has no profitable deviation. Second, anticipating the equilibrium play in the second stage, either firm has no profitable deviation in its bidding strategy, that is, $\{b_H, b_L\}$ is an equilibrium in the first stage bidding competition. Following a similar approach to that of the baseline model, we can derive the equilibrium mixed-strategy pricing and the corresponding expected profits of both firms, and then compare their bidding amounts.

The price competition in the second stage follows a similar pattern as in the baseline model, except that now firms have to take account of non-shoppers' reaction when setting the highest price they may charge. In particular, when H wins the first position, firms' equilibrium pricing strategies share a similar format as before but with a different price range:

$$\begin{aligned} F_H(p) &= \begin{cases} \frac{p-m}{p-c} & p \in [m, r) \\ 1 & p = r \end{cases} \\ F_L(p) &= \begin{cases} \frac{p-m}{(1-\alpha)p} & p \in [m, r) \\ 1 & p = r \end{cases} \end{aligned} \tag{3.7}$$

where r is the reserve price for consumers with a positive search cost defined as $r = \min\{\frac{1-\alpha}{1-\alpha+\alpha\ln\alpha}k, w\}$, and $m = \max\{\alpha r, c\}$.[9] Similarly, the expected sales profits of both firms in this scenario can be written as $\pi_H^1 = m$ and $\pi_L^2 = (1-\alpha)(m-c)$.

When L wins the first position, the reserve price of those non-shoppers becomes $r' = \min\{\frac{1-\alpha}{1-\alpha+\alpha\ln\alpha}k + c, w\}$, and the equilibrium pricing strategies can be written as

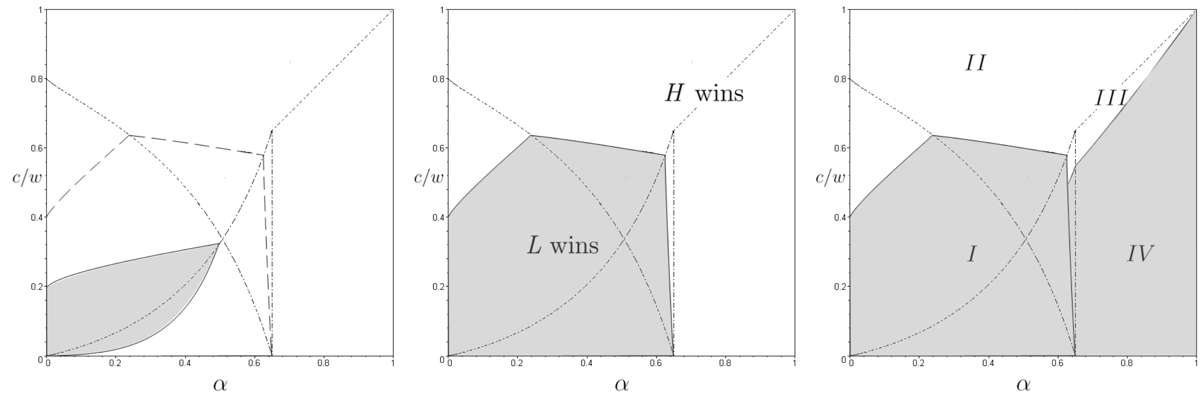
$$\begin{aligned} F_H(p) &= \tfrac{p-c-\alpha(r'-c)}{(1-\alpha)(p-c)} && p \in [c+\alpha(r'-c), r'] \\ F_L(p) &= \begin{cases} \frac{p-c-\alpha(r'-c)}{p} & p \in [c+\alpha(r'-c), r') \\ 1 & p = r' \end{cases} \end{aligned} \tag{3.8}$$

In this case, H achieves an expected profit level of $\pi_H^2 = (1-\alpha)(c+\alpha(r'-c))$, and L attains $\pi_L^1 = \alpha(r'-c)$.

We summarize the equilibrium bidding outcome and price dispersion in Figure 3.7 (with arbitrarily $k = \frac{1}{5}w$). As we can see, the equilibrium outcomes remain unchanged in pattern. The prominent position is always profitable for the low-type firm, while the high-type firm does not value the prominent position sometimes (shadowed region in Figure 3.7(a)). Only when both α and c are small (the shadowed region in Figure 3.7(b)) can the low-type firm win the first position. The expected price from the first position is higher than that from the second one, unless the cost advantage is overwhelming (the unshadowed region in Fig.3.7(c)).

When considering consumers' sequential search strategies, especially when non-shoppers' search costs are not too high, the firm at the prominent position no longer fully exploits them by setting the upper bound of the price support equal to consumers' total surplus w. The reason is simply that the firm cannot afford to lose the entire market. By charging an upper bound price as high as w, the firm would not only lose those shoppers but also invite non-shoppers to conduct an additional search and lose them as well. Realizing this, the firm with location advantage adjusts its price to retain those consumers with relatively high search costs, by setting the upper bound of price support equal to non-shoppers'

[9] In the case when $m = c$, the non-shoppers' actual reserve price r'' could be different from r, but $r \leq r'' < c/\alpha$ so that it does not affect the equilibrium outcomes.



(a) Endogenous Valuation (b) Bidding Outcome (c) Price Dispersion

Figure 3.7: Endogenous Sequential Search: $k = \frac{1}{5}w$

reserve price to keep them from further searching. Correspondingly, the firm at the less prominent position adjusts its price as well, to compete for the shoppers who sample both positions. Such auto-adjustment of pricing lowers the equilibrium prices from the two positions simultaneously, with the relative price pattern unchanged; so is the relative equilibrium profit. Naturally, as the comparative result, both the equilibrium bidding outcome and the comparison of price expectation exhibit no substantial change in pattern.

The analysis and results on consumer sequential search are also consistent with recent empirical findings. For example, Kim et al. (2010) estimate an elegant structural model of consumer sequential search using online product search data from Amazon.com and show that consumers have different search costs and high-cost consumers perform very limited search.

3.5 Extension and Discussion

3.5.1 External Information Channels

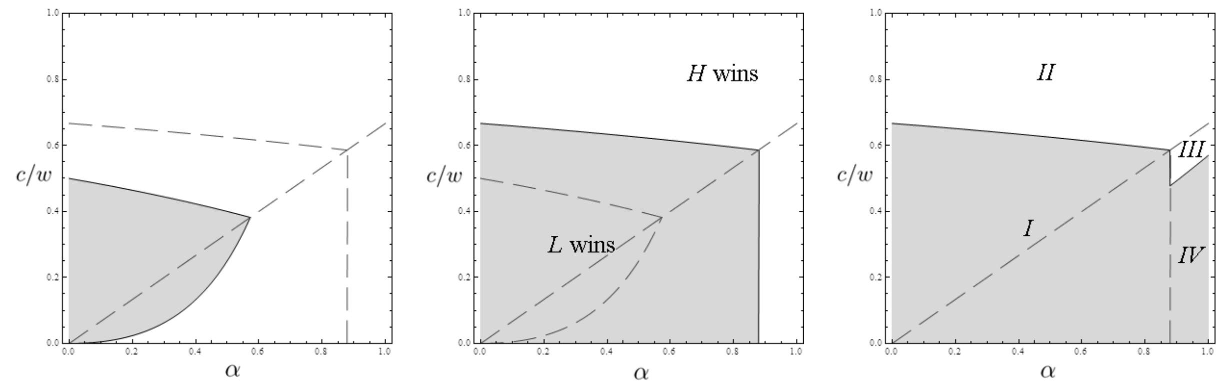
In the baseline model, we assume that all consumers obtain the product information from search advertising. Now we relax this assumption and consider the case where consumers can obtain product information from other channels. In particular, we now assume that among all consumers (with total mass 1), only $1 - M$ $(0 < M < 1)$ of them obtain price information from search advertising (i.e., from the two advertising positions considered here). The other M portion of consumers obtain price information from external channels (e.g., newspapers, television) and are assumed to be aware of both firms' product information. In fact, we can also consider different information coverage rates of firms in the outside channels and further consider overlap of information coverage between different channels, which can be shown not to affect the qualitative results. Firms charge the same price to both the search advertising and outside markets. (If the pricing decisions are made separately, then these are essentially separate markets.) In sum, $M + (1-M)(1-\alpha)$ of consumers (i.e., consumers from the outside channels plus shoppers in the search market) are informed of both firms' prices and purchase from the one offering a lower price, whereas $(1-M)\alpha$ of consumers (i.e., non-shoppers in the search market) sample the firm at the first position only and purchase from there (if the price does not exceed w).

Firms' pricing strategies can be derived accordingly. For example, when H wins the first position, the equilibrium pricing is as follows:

$$\begin{array}{l} F_H(p) = \begin{cases} \frac{p-\underline{p}}{p-c} & p \in [\underline{p}, w) \\ 1 & p = w \end{cases} \\ F_L(p) = \begin{cases} \frac{p-\underline{p}}{[(1-M)(1-\alpha)+M]p} & p \in [\underline{p}, w) \\ 1 & p = w \end{cases} \end{array}, \tag{3.9}$$

where $\underline{p} = \max\{(1-M)\alpha w, c\}$. When H wins, firms' expected sales profits are $\pi_H^1 = \underline{p}$ and $\pi_L^2 = [(1-M)(1-\alpha)+M]\left(\underline{p}-c\right)$; when L wins, firms achieve profit levels $\pi_L^1 = (1-M)\alpha(w-c)$ and $\pi_H^2 = [(1-M)(1-\alpha)+M][(1-M)\alpha(w-c)+c]$. Firms bid $b_i = \frac{\max\{\pi_i^1-\pi_i^2,0\}}{1-M}$ per click and are ranked based on the score $s_i = \max\{\pi_i^1 - \pi_i^2, 0\}$, where $i \in \{H, L\}$. Notice that when $M = 0$, all results reduce to those from the baseline model.

Figure 3.8 illustrates the results when $M = \frac{1}{3}$. As we can see, the high-type firm's endogenous valuation, the bidding outcome, and the spatial price dispersion all follow patterns similar to those in the baseline model. For example, H achieves higher profit in the second position than the first one in the shadowed region in Figure 3.8(a), and the expected price from the first position is lower in the unshadowed region in Figure 3.8(c).



(a) Endogenous Valuation (b) Bidding Outcome (c) Price Dispersion

Figure 3.8: External Information Channels: $M = 1/3$

An interesting aspect worth noting is that as M increases, the regions where H does not value the first position and where H fails to outbid L both expand rather than shrink (compared to the baseline model where $M = 0$). It implies that when the search advertising market is only part of the entire product market, as it is in reality, the trade-off pointed out in this study is even more salient. Recall that two counterbalancing effects determine the profitability of winning the prominent position for the high-type advertiser: capturing non-shoppers when winning the position versus benefiting from a higher premium charged to shoppers when letting the weaker competitor take the location advantage. When M increases, the relative size of the search market decreases and thus the loss from losing the non-shoppers decreases, which reduces the relative significance of the first effect. Meanwhile, the extra demand from the external market increases the benefit from weakening the price competition and raising the equilibrium prices, which enforces the second effect. As a result, the first position actually becomes less appealing to the high-type firm.

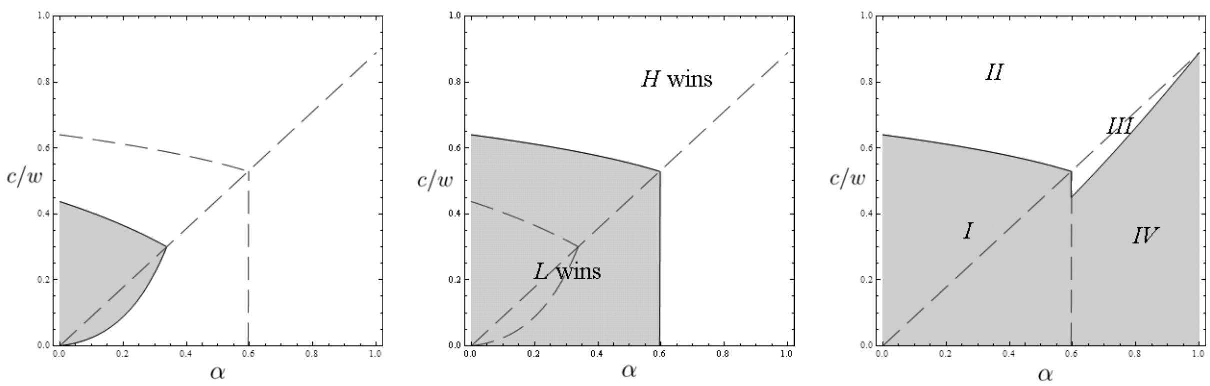
3.5.2 Heterogeneous Consumer Preference

We now relax the homogeneous product assumption and allow consumers to have heterogeneous preferences. Starting from the baseline model, similar to Narasimhan (1988), we now assume among all consumers (with total mass 1), t_1 of them are loyal to H's product, t_2 of them are loyal to L's product, and the rest $1 - t_1 - t_2$ $(0 < t_1 + t_2 < 1)$ do not have a particular preference and purchase from the firm offering a lower price. Assume firm i's loyal customers visit firm i's position directly and buy if the price does not exceed w ($i \in \{H, L\}$). The rest of the consumers follow the same search pattern: α of them are non-shoppers and $1-\alpha$ are shoppers.

Following a similar analysis, we can derive the equilibrium outcome. For example, when H wins, the pricing strategies are:

$$\begin{array}{l} F_H(p) = \begin{cases} \frac{[(1-t_1-t_2)(1-\alpha)+t_2]\left(p-\underline{p}\right)}{(1-t_1-t_2)(1-\alpha)(p-c)} & p \in [\underline{p}, w) \\ 1 & p = w \end{cases} \\ F_L(p) = \begin{cases} \frac{(1-t_2)\left(p-\underline{p}\right)}{(1-t_1-t_2)(1-\alpha)p} & p \in [\underline{p}, w) \\ 1 & p = w \end{cases} \end{array}, \tag{3.10}$$

where $\underline{p} = \max\{\frac{(1-t_1-t_2)\alpha+t_1}{1-t_2} w, \frac{t_2}{(1-t_1-t_2)(1-\alpha)+t_2}(w-c)+c\}$, and firms' expected profits are $\pi_H^1 = (1-t_2)\,\underline{p}$ and $\pi_L^2 = [(1-t_1-t_2)(1-\alpha)+t_2]\left(\underline{p}-c\right)$. The per-click bids are $b_H = \frac{\max\{\pi_H^1-\pi_H^2,0\}}{1-t_2}$ and $b_L = \frac{\max\{\pi_L^1-\pi_L^2,0\}}{1-t_1}$, whereas the scores are still $s_i = \max\{\pi_i^1 - \pi_i^2, 0\}$ ($i \in \{H, L\}$), as is shown before. Figure 3.9 illustrates the results when $t_1 = t_2 = 0.1$, showing that the pattern remains unchanged.



(a) Endogenous Valuation (b) Bidding Outcome (c) Price Dispersion

Figure 3.9: Heterogeneous Consumer Preference: $t_1 = t_2 = 0.1$

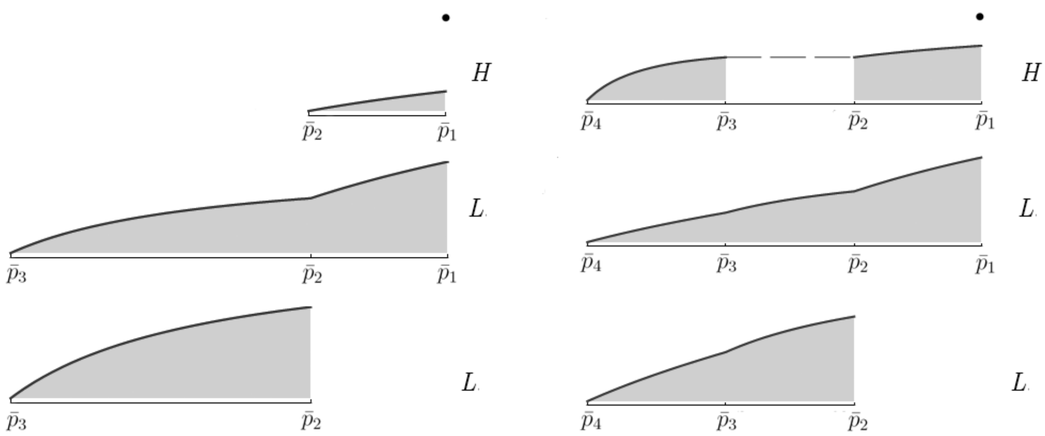
Consumers' heterogeneous preferences affect the results only to the extent that they change the total size of the market for which firms compete. Nevertheless, as long as there is still a certain portion of consumers willing to switch between products, the aforementioned trade-off remains. Therefore, the results of interest change only quantitatively rather than qualitatively.

3.5.3 Multiple Competing Firms

Although mainly based on duopoly analysis, the results actually hold beyond the case of only two firms. In this section, we consider a case of three competing firms to show that the main results can be extended to the oligopolistic setting.

Extending the baseline model, we now consider three firms with different production costs c_1, c_2, and c_3. We consider a simple case in which $c_1 = c_2 > c_3$. The more general case that $c_1 \geq c_2 \geq c_3$ would add further technical discussion, whereas the qualitative results of interest can be expected to hold. Without loss of generality, we normalize $c_3 = 0$ and denote $c_1 = c_2 = c$ $(0 < c < w)$. Again, we call the low-cost firm the *high type*, or H, and the two high-cost firms the *low type*, or L. There are three advertising positions with different prominence levels, reflecting consumers' search ordering. Similarly, assume that all consumers (with total mass 1) start searching from the first position; among them, α_1 stop and purchase from the first position if the price does not exceed w. The rest $1 - \alpha_1$ continue searching and sample the second position; α_2 of them (i.e., with a total mass $\alpha_2 (1 - \alpha_1)$) stop searching after knowing the first two firms' prices and buy from the one offering the lower price. The rest $(1 - \alpha_1)(1 - \alpha_2)$ are shoppers, who sample all three firms and buy from the one offering the lowest price. For ease of exposition, we let $\alpha_1 = \alpha_2 = \alpha$ $(0 < \alpha < 1)$. The analysis and results can be naturally extended if α_1 and α_2 take different values. The other settings follow the baseline model.

The price competition among three firms becomes much more complex than the duopoly case. A complete description of the equilibrium pricing is detailed in Appendix C.3. Figure 3.10 illustrates two equilibrium price patterns that exhibit interesting features. Figure 3.10(a) depicts the cumulative distribution functions for the equilibrium pricing when H stays in the first position and $\frac{c}{w} < \frac{\alpha^2}{1+\alpha(1-\alpha)}$. Notice the stair shape of the equilibrium price supports. Overlap of price supports exists only between directly adjacent



(a) $\alpha = 0.4$, $c = 0.1$, $w = 1$ (b) $\alpha = 0.4$, $c = 0.3$, $w = 1$

Figure 3.10: Equilibrium Pricing with Three Firms

firms. There is no direct price competition between firms placed far from each other. The localized competition reflects the subtle interaction between cost advantage and location advantage. With limited cost advantage (i.e., $\frac{c}{w}$ is relatively small), H would rather take advantage of its good location by charging high prices than enter the competition for shoppers with very low prices. Likewise, getting into the high-price range is not profitable for L in the third position, which has neither cost nor location advantage. In contrast, Figure 3.10(b) represents the case where H stays in the first position and $\frac{\alpha^2}{1+\alpha(1-\alpha)} < \frac{c}{w} < \min\{\alpha, \frac{1-\alpha(1-\alpha)}{[1+\alpha(1-\alpha)]^2}\}$. The probability mass near the lower bound of H's price support (which does not appear in Figure 3.10(a)) indicates that with considerable cost advantage, H is willing to compete for shoppers with more competitive prices. This unique pricing pattern that involves segmented price supports and localized price competition is absent in the typical price competition literature.

Table 3.2 summarizes firms' equilibrium profits from price competition under different scenarios. If we compare H's profits in different positions, similar results arise in the three-firms case: When evaluating endogenously in the product market competition, a less prominent position might not mean less profit. As is shown in Figure 3.11, in region I, the third position generates the highest equilibrium profit for H among all three positions (i.e., $\pi_H^3 > \pi_H^1 > \pi_H^2$); in region II, the third position is more profitable than the second one (i.e.,

Table 3.2: Equilibrium Profits from Price Competition in the Case of Three Firms

	$H-L-L$	$L-H-L$	$L-L-H$
When $\frac{c}{w} < \frac{\alpha^2}{1+\alpha(1-\alpha)}$			
Profit in 1st Position	αw	$\alpha(w-c)$	$\alpha(w-c)$
Profit in 2nd Position	$\alpha(1-\alpha)\left[\frac{w}{1+\alpha(1-\alpha)}-c\right]$	$\alpha(1-\alpha)\frac{w+(1-\alpha)c}{1+\alpha(1-\alpha)}$	$\alpha(1-\alpha)(\bar{p}_2-c)$
Profit in 3rd Position	$(1-\alpha)^2\alpha\left[\frac{w}{1+\alpha(1-\alpha)}-c\right]$	$(1-\alpha)^2\left[\frac{\alpha w-c}{1+\alpha(1-\alpha)}\right]$	$(1-\alpha)^2\bar{p}_3$
When $\frac{\alpha^2}{1+\alpha(1-\alpha)} < \frac{c}{w} < \alpha$			
Profit in 1st Position	αw	$\alpha(w-c)$	$\alpha(w-c)$
Profit in 2nd Position	$(1-\alpha)(\alpha w-c)$	$\alpha(1-\alpha)\frac{w+(1-\alpha)c}{1+\alpha(1-\alpha)}$	$\alpha(1-\alpha)(\bar{p}_2-c)$
Profit in 3rd Position	$(1-\alpha)^2(\alpha w-c)$	$(1-\alpha)^2\left[\frac{\alpha w-c}{1+\alpha(1-\alpha)}\right]$	$(1-\alpha)^2\bar{p}_3$
When $\frac{c}{w} > \alpha$			
Profit in 1st Position	c	$\alpha(w-c)$	$\alpha(w-c)$
Profit in 2nd Position	0	$(1-\alpha)c$	$\alpha(1-\alpha)(\bar{p}_2-c)$
Profit in 3rd Position	0	0	$(1-\alpha)^2\bar{p}_3$

Note:
1. $H-L-L$ represents the case when H stays in the first position.
2. $\bar{p}_2 = \frac{-(1-\alpha)(1-2\alpha)c+w+\sqrt{5(1-\alpha)^2c^2-2(1-\alpha)(1-2\alpha)wc+w^2}}{2[1+\alpha(1-\alpha)]}$.
3. $\bar{p}_3 = \alpha(\bar{p}_2-c)+c$.

$\pi_H^1 > \pi_H^3 > \pi_H^2$). In other words, in the shadowed region, the "worst" position actually outperforms a "better" position for the high-type firm. In addition to the endogenous valuation, similar results on the equilibrium bidding outcome and spatial price dispersion pattern can be derived as well (see Appendix C.4 for details).

In the case of multiple firms with different competitive strength competing against each other, when we endogenously investigate the price competition, the weaker firms tend to charge less competitive prices once placed in the good positions, which leaves a higher profit margin for the stronger firm in a lower position and hence reduces its bidding incentive. Therefore, the trade-off of interest remains, and similar results can be derived.

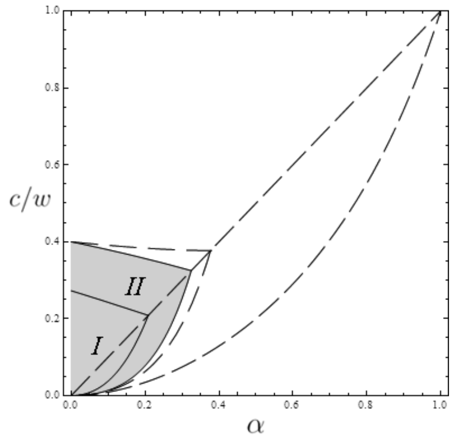


Figure 3.11: Endogenous Valuation in the Case of Three Firms

3.5.4 Supportive Observations

While the new perspective on location choice and pricing decisions proposed in this study might easily be neglected by some marketing managers in practice, there are many empirical observations consistent with the results from our modeling analysis as well. We provide some examples in this section.

To investigate advertisers' bidding behaviors in reality, we track the actual sponsored ranking results from the leading online search engine *Google*. A program was designed to automatically enter search queries using the given keywords every five minutes and to record the ranks of targeted firms' sponsored links for three consecutive weeks starting from the noon on May 18, 2010. Notice that in addition to the regular sponsored links on the right side of the web page, *Google* also provides premium sponsored positions in a highlighted region right above the general search results, which are much more noticeable and usually much more costly. We thus rank the premium sponsored positions higher than the regular ones. For example, if there are two premium positions, then the first regular sponsored link is ranked number three. Keywords are chosen to fit our model setting as closely as possible. Table 3.3 summarizes the statistics of the data recorded.

The observations shown in Table 3.3 can be well interpreted by our model results. Both textbooks and photo prints are a relatively standard product or service, so price

Table 3.3: Summary Statistics of the Sponsored Ranking Data (Total time periods: 6048)

	Appearance	*Mean Rank*	*St. Dev.*
Keyword: *Textbooks*			
Textbooks.com	5993	1.065076	0.260501
Amazon.com	6040	2.443543	0.743103
Keyword: *Online Photo Print*			
YorkPhoto.com	6042	1.002979	0.070407
Shutterfly.com	6040	2.395530	0.717865
Keyword: *Car Rental*			
Budget	6043	2.098957	0.922594
Avis	6040	2.488079	1.530998
Enterprise	5466	4.600256	2.893965
Keyword: *Dell Laptop*			
Dell	6042	1.000331	0.025730
Staples	5715	4.280315	1.220876

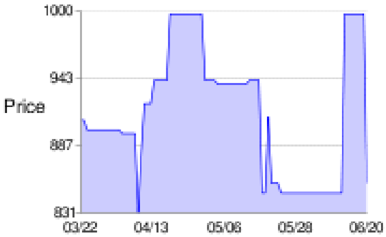
would be the primary consideration. *Textbooks.com* is a website selling new and used textbooks. In contrast, *Amazon*, as the largest online bookstore and marketplace, could have lower average marginal costs, probably owing to economy of scale, better managed information systems, or greater bargaining power over the supply chain. Thus, we might consider *Amazon* as the high-type firm. Similarly, compared to the NASDAQ-listed leading digital photo service company, *Shutterfly*, *YorkPhoto.com* is smaller in scale and probably weaker in competitive strength. Nevertheless, given the highly standardized products, it can be expected that the cost differences should be small in both cases. Since the price quoting process is relatively straightforward in both cases, consumers can easily compare prices, which could result in a small α value. As is shown, when α and c are small, the low-type firm may have higher bidding incentive, which is reflected by the consistently higher ranks of both *Textbooks.com* and *YorkPhoto.com*.

In the car rental example, North America's largest rental car company, *Enterprise*, stays at a lower sponsored rank in general, which can be interpreted similarly as the previous two examples. Interestingly, the high variance in its ranking is worth attention. Because the search process is more complex than the previous two examples, which potentially corresponds to a higher α value, and because the cost advantage of the market leader versus smaller firms might also increase, therefore, by our results, with an increased α and/or c, the high-type firm's bidding incentive might increase accordingly. In fact, from the collected data, *Enterprise* wins the first position about one-fourth of the time and stays in the fifth or lower positions more than half of the time. It fits our results in the case of multiple competing firms: With considerable α and/or c values, H may adopt mixed-strategy bidding and either outbids both L firms or stays in the lowest position (see Appendix C.4 for details).

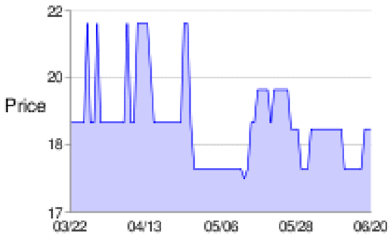
The fourth example corresponds to the case of dominating cost advantage. Because *Google* restricts the bidding to brand names, only authorized firms can bid for keywords containing particular brand names. *Staples* is *Dell*'s authorized retailer and meanwhile is the competing channel of *Dell*'s direct marketing. When they compete in search advertising, their decisions may be considered as roughly independent. Producing and selling the same laptops, *Dell* undoubtedly possesses significant cost advantage. Similar to the model results, *Dell* tightly holds the best advertising position, with no exceptions.

There is also evidence supporting the results on equilibrium pricing. In addition to the aforementioned literature, websites that watch the real-time product prices on *Amazon.com* find significant levels of temporal price fluctuation in various product categories. Figure 3.12 shows some findings from one such website. The spatially differentiated price expectation pattern can also be examined using the examples given. For instance, *Amazon* generally offers more competitive textbook prices and *Dell* online store is generally believed to sell cheaper *Dell* laptops than other retailers,[10] which are consistent with the model predictions.

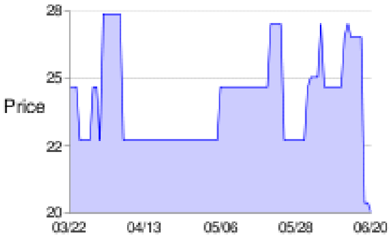
[10]A random price comparison on June 9, 2010, finds that the classical microeconomics textbook, *Microeconomic Theory,* is sold at \$114.94 on *Amazon* but \$118.68 on *Textbooks.com.* Another random price check on June 27, 2010, finds that the Dell Inspiron 15" laptop is sold at \$639.98 on *Staples.com*, but it can be bought from *Dell.com* at \$584.99 with the same configuration.



(a) Electronics: *Casio Exilim Camera*



(b) Book: *Too Big to Fail*



(c) DVD: *Toy Story 2*

(d) Tool: *String Trimmer Spool*

Figure 3.12: Daily Prices of Different Products on *Amazon*
(Source: http://www.frozenwarrior.com/~pricewatch)

In addition, other model results are also supported by real-world data. For example, recall that, as is shown in Figure 3.4, the bidding competition is the keenest when α is very large and c is quite small. Interestingly, a website called CyberWyre (www.cwire.org) keeps an updated list of the highest paying search terms. Currently, the most expensive search terms are mesothelioma-related lawsuits, which can cost as high as \$69.1 per click. As reported in a *New York Times* article (Liptak, 2007), mesothelioma cases are relatively routine and "settle rather easily," which indicates negligible cost differences. In contrast, the search process can be quite consuming because lawyers "will steer you into highly tendentious information" so as to capture these clients, which results in a high α value. As a result, law firms "compete on *Google* instead of competing on price," which fits the model results that firms bid aggressively and charge a high price upon winning under such circumstances.

3.6 Conclusion

When marketing managers deal with location choice, such as competing for online advertising slots, understanding the value of a premium location is a fundamental issue. Only if they comprehend the value difference between locations can managers optimally allocate their spending to achieve the best possible marketing results. In this study, we investigate the value of a prominent advertising position endogenously in the context of price competition among asymmetric advertisers in the search advertising setting. We examine the equilibrium outcome of the bidding competition, as well as the resulting price dispersion pattern in various scenarios.

Compared with the existing literature, we are the first to illustrate that, in search advertising, the value of the advertising slots should be determined endogenously in price competition rather than taken for granted exogenously. For a particular advertiser, the per-click value, instead of being fixed, could vary across different slots depending on the competitor it faces and how consumers search. A prominent advertising slot is not always desirable, even if it is cost-free. We identify a sophisticated pattern of price dispersion resulting from the unique features of online consumer search behavior. This work is among the very few studies focusing on the asymmetric competition among advertisers on the issue of price advertising.

Our analysis has several implications for marketing managers. We underscore the fact that advertisers' willingness-to-pay for prominent locations should not be determined in isolation. In-depth investigation of a firm's relative competitive strength within the industry are crucial in determining firms' advertising spending. Firms in different competitive situations should tailor their advertising strategies accordingly. In particular, a firm with competitive advantage in some cases could even be better off by staying at a less prominent position and by pricing properly to soften price competition. Such competence-dependent evaluation calls for coordination and communication between marketing teams and other business functions (e.g., production and sales) in a company.

Likewise, thorough market investigation regarding consumer search behavior is indispensable. Consumer search patterns may vary across different products (e.g., books vs. computers) or across different time periods (e.g., weekdays vs. weekends). Thanks to the advance of information technology, the investigation can be conducted at a lower cost and the information collected more easily than ever before (e.g., search engines usually track the clicks of sponsored links at different ranks for different keywords).

Most importantly, our analysis provides the rationale for firms to determine their spending as they compete for a prominent advertising position. The rule of thumb is that both the relative competitive strength and prominence difference matter in determining firms' bidding strategies. Firms that have a competitive advantage, when neither their competitive advantage nor the location prominence difference is salient, should forgo the most prominent slots and leverage their revenue instead by lowering the price to capture consumers. In contrast, disadvantaged firms should bid aggressively in this scenario to reap the benefit of the prominent position. When either the competence difference or the prominence difference is significant, disadvantaged firms should avoid being too ambitious and over-investing in the bidding competition.

The price dispersion patterns derived from our model are also of interest to consumers. Because of the two-dimensional price dispersion, in general there is no straightforward way to find the lowest price in a one-shot search. For consumers who have low search costs, we recommend conducting a thorough search. For those who are not willing to search a lot, sampling only the prominent position might be wise because it might sometimes offer good deals as well.

While we use online search advertising as the setting for our discussion, our model and analysis might also apply to other settings involving location acquisition and price competition. This is because the rank of an advertising slot in the online world is similar to the degree of prominence of business locations in the physical world—from stores in a shopping mall to gas stations along a highway to shelf space in grocery stores. Take slotting in supermarkets as an example. It is commonly believed that product location on the shelf has an important effect on sales, and a central location at eye level is most desirable (Dreze et al., 1994). Given the significant difference in prominence and the scarcity of prominent shelf space relative to the number of products, firms compete intensely for shelf positions by paying various forms of "slotting allowances," which are lump-sum advance payments made to retailers by manufacturers for stocking their products on the shelf. The results and insight delivered in this study also shed light on slotting allowances, in that manufacturers compete for prominent shelf positions and battle for consumers via pricing, resembling the search advertising case in many ways.

This work triggers interesting directions for future research. Here, firms' valuation of the prominent position is endogenized in the pricing competition; this can be viewed as an example of an unexplored class of auctions in which an object's value to a particular bidder depends on its competitors. The study of such auctions becomes even more exciting if extended to a general case in which heterogeneous firms compete for multiple display positions, combining multiple-object auctions and asymmetric oligopoly price competition together.

Chapter 4

Interplay Between Organic Listing and Sponsored Bidding in Search Advertising

4.1 Introduction

Search advertising, in which advertisers bid for *sponsored* advertising slots listed on a search engine results page (SERP) alongside a list of *organic* (non-sponsored) links, has proven itself to be a successful revolution of traditional online and offline advertising. Internet search-related advertising is predicted to generate annual revenue over $45 billion worldwide by 2011, becoming the leading advertising medium (VSS, 2007). The huge industrial success has attracted increasing academic interest, which includes recent theoretical studies on advertisers' sponsored bidding strategy and the optimal auction mechanism to sell the sponsored slots, and some empirical work investigating factors that affect the profitability of sponsored advertising. Nevertheless, the organic list, despite its being the origin and the major information source of search advertising, has been generally neglected in the literature. This study aims to systematically analyze the effects of organic listing as a competing information source on the advertising competition (i.e., sponsored bidding) and the outcome performances in search advertising.

Two features deserve special attention in studying the role of organic listing. One is the unique information structure associated with a SERP; the other is the characteristics of the commonly used organic ranking mechanism.

In response to each user query of a particular keyword, the search engine returns a SERP that contains hyperlinks to websites related to the keyword. In practice, major search engines (e.g., Google, Yahoo!, and Bing) organize the SERP in a similar way. Two lists of links are paralleled: A list of non-sponsored links, or *organic* links, is placed

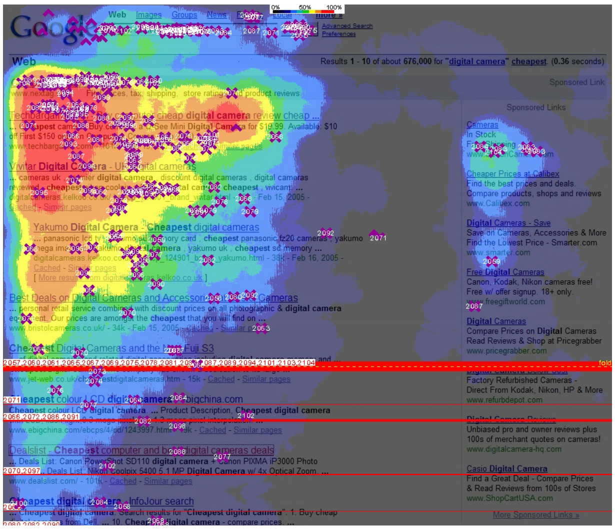


Figure 4.1: Eye movement of users viewing Google pages (Hotchkiss et al., 2005)

in the wide column on the left, and a narrow column on the right (and sometimes a highlighted area on the top as well) contains the list of *sponsored* links. Organic links are ordered based on search engines' proprietary ranking algorithm, and the ordering typically reflects different links' relative relevance to the keyword. The sponsored list is composed of advertising slots for sale. They are usually sold via auction, in which the bidder with the highest bid (or score) wins the first sponsored slot, the second highest wins the second, and so on.

One distinction of the co-listing structure is that it creates two lists competing with each other for consumer attention. Figure 4.1 shows the result from an experiment tracking eye movement of users viewing Google search result pages (Hotchkiss et al., 2005). The reddest region indicates the highest attention level (i.e., 100%). As we can see, the top organic links attract the most attention (e.g., the top three organic links are viewed by almost all experiment participants), while the top sponsored link attracts a certain level of attention but could be less significant compared to its organic counterparts (e.g., the first sponsored link is viewed by about half of the participants). Notice that those merchant websites interested in sponsored advertising may also appear in the organic list and thus could get significant attention from the organic list without paying anything. In this sense, the organic list not only competes for consumer attention but also plays a dominating role in such competition. Then why would advertisers placed at prominent positions in the organic list still be willing to spend money on sponsored bidding? By creating a competing list to its revenue source (i.e., the sponsored list), is the search engine jeopardizing its own revenue?

In addition to the information structure of SERP, the organic ranking rule is another key element. Although kept private in most cases, organic ranking rules, such as Google's PageRank-based ranking rule, are commonly believed to fairly reflect relative relevance or popularity of different websites by utilizing the inter-linking structure of websites and many other factors. Economic analysis also validates that websites' relative quality or relevance is aligned with their equilibrium number of incoming links, which is consistent with the essence of the typical link analysis algorithm (e.g., PageRank algorithm) used by search engines (e.g., Katona and Sarvary, 2008). In other words, a website with greater popularity or relevance is generally given a better position with higher prominence in the organic list, and vice versa. Among those merchant websites that are potential sponsored advertisers, the typical organic ranking mechanism tends to favor the leading firms by giving them higher organic ranking and allocating them a greater level of prominence. Under such asymmetric allocation of the organic prominence resource, how different advertisers react in the sponsored bidding and what the bidding outcome is become subtle questions. Will the leading firms take advantage of their preempted prominence advantage and patronize the sponsored advertising actively, or might small firms bid aggressively as a fight back for the disadvantage in organic listing?

Directly related to the organic ranking mechanism, another issue of interest is whether typical organic ranking promotes sales diversification. It has been well documented that in the ecommerce environment, millions of small firms and individual sellers are able to survive and flourish, and the overall sales diversity is greatly increased, all of which add to the richness and diversity of the online community and become the spirit of the dot-com era. As the leading online marketing medium and major information gatekeeper, search engines recognize their role in promoting small advertisers and increasing the sales diversity. For example, Eric Schmidt, CEO of the dominating search giant Google, describes the company's mission as "serving the long tail."[1] Nevertheless, given the fact that large-scale mainstream firms are normally given prominent positions in the organic list, is the typical organic ranking mechanism really promoting the "tails," or just making the big bigger? If the latter, it would be appealing to think of any possible improvement of organic ranking to better achieve the goal of "serving the tails."

Motivated by these intriguing issues, this study intends to capture the unique feature of the co-listing structure (i.e., organic list attracts most attention) and the essential characteristics of the organic ranking mechanism (i.e., organic ranking aligns with relative popularity), so as to explicitly address the following research questions related to organic listing:

- What are advertisers' bidding incentives for sponsored slots in the presence of the organic listing? How do such incentives differ across various advertisers, and what is the expected bidding outcome?
- How does organic listing affect the outcome performance, particularly social efficiency and search engine benefit? How does organic listing affect the resulting sales diversity, and how well does it serve the tails?
- In certain cases, is there any possible way to improve the organic ranking mechanism to benefit both the search engine and the online society?

We consider a game-theoretic model in which firms in different organic slots compete for sponsored slots via auction and then compete for consumers in price after getting different sponsored slots. Firms are *asymmetrically differentiated* in terms of market preference. The mainstream firm is preferred by the majority of consumers in the market, while the niche firm is preferred by a small portion of the market. We model the organic ranking outcome in a way that the leading firm in terms of market preference gets a

[1] http://longtail.typepad.com/the_long_tail/2005/05/google_longtail.html

top organic position with high prominence advantage. Consistent with the experimental findings, we assume that the top several organic positions attract a fairly high level of attention while the first sponsored link attains less but still reasonable attention level. Prominence decreases rapidly from the top downward in both organic and sponsored lists. We investigate the equilibrium outcome of the bidding competition, in which the value of sponsored slots is *endogenously* determined in the pricing competition. We analyze the effects of organic listing on equilibrium outcomes (including social welfare, sales diversity, and search engine benefit) by comparing with a benchmark case with the sponsored list only (without the organic list). We then propose possible improvement of the organic ranking mechanism so that search engine benefit, welfare, and diversity can all be increased in certain cases.

In analyzing the bidding competition, we identify two interacting effects that drive advertisers to compete for sponsored listing in the presence of the organic list, namely, the *promotive* effect and the *preventive* effect. The promotive effect, which means a firm can promote its exposure by winning a prominent sponsored slot, decreases when the firm's organic prominence increases; in contrast, the preventive effect, that a firm can prevent its competitors from increasing their prominence by occupying the prominent sponsored position, increases as the firm gets a better position in the organic list. We find that firms' equilibrium profit functions are submodular, that is, the marginal benefit of improving sponsored prominence changes in the same direction as firms' relative competitive strength. As a result, we show that when the competing firms are relatively comparable to each other, the disadvantageous one bids aggressively and wins the prominent sponsored slot; when the market preference is highly asymmetric, the leading firm outbids its competitor, occupying prominent positions in both organic and sponsored lists.

Compared to the case with no organic list, organic listing subsidizes the leading advertisers in prominence for free to dilute their sponsored bidding incentive and to adjust the competence difference among advertisers. In general cases with moderate levels of market asymmetry, the effects of such subsidy are two-fold. On the one hand, the co-listing structure induces weak advertisers to win better sponsored positions, which effectively increases their exposures without impairing the objectivity of the organic list. As a result, while keeping general search engine users satisfied, the co-listing structure improves the surplus of potential consumers (who are looking for product information), overall social welfare, and sales diversity. On the other hand, the free exposure from organic listing reduces advertisers' sponsored bidding incentives, which reduces the search engine's direct revenue. In this sense, organic listing serves as a balance between short-term and long-term benefit—sacrificing short-term revenue to enhance consumer surplus, total welfare and sales diversity, which could lead to better long-term growth.

While organic listing's beneficial effects function well in general, they may malfunction when coming to highly asymmetric competition among advertisers. For small firms at the very tail facing strong competitors, their bidding incentive would be too low to win a prominent sponsored position. As a result, the search engine not only bears direct revenue loss but also fails to induce structural improvement of consumer surplus, social welfare and sales diversity. To explore possible ways of mitigating such shortcomings and better serving the tail, we generalize the typical popularity-based *pure* organic ranking and propose a *mixed* organic ranking mechanism, which probabilistically places less popular websites to a prominent position in the organic list rather than ranking strictly based on popularity. We show that introducing mixed organic ranking in a highly asymmetric market could improve the search engine's short-term revenue, as well as consumer surplus, social welfare and sales diversity concurrently.

In addition to its substantive contribution to the literature on search advertising, this study also has notable theoretical contributions. Compared to traditional economics of advertising literature and recent studies related to search advertising, the substantially new understanding added by this work, from theoretical perspective, lie in at least two aspects. First, we view organic list as an extra information source in addition to the advertising channel (i.e., the sponsored list), and we study how such a unique information structure affects advertisers' advertising strategies as well as the equilibrium outcomes.

Second, we consider advertising resources as differentiated and exclusive.

Organic listing serves as a competing information source added into the advertising campaign, which brings new perspectives into informative advertising studies. From the informative advertising perspective, advertising competition is essentially competition for information coverage (Bagwell, 2007). In typical informative advertising, advertising channels function as the major information sources that convey the information of firms' existence, product details, and prices. In search advertising, however, a non-advertising channel (i.e., the organic list) coexists with the advertising channel, provides similar information, and even plays a dominating role as the major information source. Unlike sponsored positions, organic ranking and exposure levels are supposed to be out of advertisers' control and thus exogenous. Moreover, the ordering of organic links and the resulting differences in organic exposure correlate with advertisers' intrinsic competence (e.g., relative popularity in market preference). Such a unique source of information, which any traditional advertising channel can hardly resemble, naturally affects advertisers' advertising strategies in a distinctive way, as is elaborated in this study.

The advertising resource is *exclusive* in search advertising by nature, as different slots have very different prominence levels and only one advertiser can stay at the most prominent advertising position. Advertisers have to compete against each other for good advertising positions, and auction is naturally introduced to sell these positions. Consequently, a small difference in competence could mean the huge difference between winning or losing the best advertising resource. This feature compels us to model the very small difference in firms' competence and to explicitly analyze the bidding outcomes. Traditional economic models of advertising, starting from Butters (1977), consider advertising technology in which advertisers independently decide advertising levels, which makes the equilibrium outcome less sensitive to the competence difference among firms. Naturally, symmetric competition remains the theme of traditional advertising literature (Grossman and Shapiro, 1984; Stegeman, 1991; Stahl, 1994). In contrast, we consider *asymmetric* competition among advertisers and incorporate asymmetric differentiation, as a combination of horizontal and vertical differentiation, into the model.

There is a large volume of literature on mechanism design and bidding strategies in sponsored auctions (e.g., Athey and Ellison, 2010; Edelman et al., 2007; Liu et al., 2010). As they consider the sponsored list only, we deepen the understanding of search advertising beyond these works by considering the interactions between the two lists. Another key distinction of our work is that most of these works treat the per-click value of a sponsored link as exogenously given, while we *endogenously* investigate the valuation of sponsored positions in price competition, connecting the product market competition with the sponsored bidding competition.

A limited number of studies focus on the role of organic listing in search advertising from different angles. Katona and Sarvary (2010) study the bidding patterns in search advertising when considering organic listing. Yang and Ghose (2010) are among the earliest to empirically investigate the potential synergistic effect between organic and sponsored links. This study complements their works by systematically examining the effects of organic listing as an additional information source on advertisers' bidding incentive for sponsored slots, search engines' revenue, consumer surplus, social welfare, and sales diversities in equilibrium.

Some recent studies on referral intermediaries also provide relevant implications. Weber and Zheng (2007) develop an elegant model of search intermediary to study firms' bidding strategies and the search engine's optimal design, considering consumers' search behavior. Their work relates to our study to the extent that with the optimal quality-weighting factor, the single list of sponsored positions considered in their model also exhibits certain features of organic listing in that the ranking partially conveys the information about advertisers' relative performances. Nevertheless, since they look at a different question and focus on the sponsored list only, their model does not capture the unique information structure under the co-listing setting and thus does not consider the interaction between the difference in organic exposure and the response in sponsored bidding, which is the focus of our study. White (2009) studies the interaction between advertising and non-advertising lists from an interesting angle: by incorporating more firms in the non-advertising list, the intermediary can improve its quality to attract more users, but more firms in the lists bring down the market price (as a result of Cournot competition), lower the advertising firms' profits, and may hurt the intermediary's revenue. Since all positions in both lists are considered the same and one firm cannot appear in both lists, there is no informative interactions between the two lists and no bidding competition among advertisers in his model.

The rest of the chapter is organized as follows. In Section 4.2 we lay out the model. Section 4.3 derives the equilibrium pricing and bidding outcome. In Section 4.4, we first set up a benchmark case with no organic list and derive the corresponding equilibrium outcome. We then compare it with the equilibrium outcome derived under the regular case as in Section 4.3, to illustrate the effects of organic listing. As such analysis reveals a potential drawback of organic listing, we propose possible directions for improving organic ranking in Section 4.5. Section 4.6 concludes the chapter with discussion on managerial implications.

4.2 The Model

We consider a search engine providing information about products. The search engine returns a SERP in response to each query of a particular keyword describing a certain type of product. Each SERP contains two lists of hyperlinks, namely, the *organic* list and the *sponsored* list. The organic list is composed of n organic links. These organic links are ranked by a proprietary algorithm designed by the search engine, in an order that reflects websites' popularity and relevance to the keyword. The sponsored list contains s sponsored links. The slots for these links are sold via auction.

Consumers' click behavior on each SERP is modeled in the following general way: When a firm's link appears in only one of the two lists (e.g., in either the ith organic slot or the jth sponsored slot), an individual consumer clicks the ith organic link with probability α_i and clicks the jth sponsored link with probability β_j. When a firm's link appears in both lists (e.g., in both the ith organic slot and the jth sponsored slot), an individual consumer clicks at least one of that firm's links with probability $1-(1-\alpha_i)(1-\beta_j)(1-\gamma_{ij})$. Without loss of generality, let $\alpha_1 \geq \alpha_2 \geq ... \geq \alpha_n$ and $\beta_1 \geq \beta_2 \geq ... \geq \beta_s$. We assume $\beta_i < \alpha_i$, as it is generally believed that, compared to organic listing, consumers have a negative bias against the sponsored list due to the aversion to advertising. We use γ_{ij} to capture the potential synergistic effect between organic and sponsored listing in terms of attracting click-throughs. As is implied by the probability expression, $\gamma_{ij} > 0$ indicates complementary effect, whereas $\gamma_{ij} < 0$ implies substitute effect and $\gamma_{ij} = 0$ means no significant synergistic effect.

We consider two competing firms selling a certain type of product in the market.[2] Their products are differentiated with *asymmetric* market preference. The firm whose products are favored by the majority of consumers or the mainstream of the market is denoted as M; the firm selling products preferred by the minority of consumers or the niche market is termed as N. Assume the two firms have the same production cost, which is normalized to zero.

There is a continuum of consumers with unit mass in the market. Each consumer has a unit demand of the product. Consumers differ in their taste. The majority of the market, with a portion $1-\theta$ ($0 < \theta < \frac{1}{2}$), prefer firm M's product to N's, while the others, with a proportion θ, prefer firm N's product. We sometimes call the former M-type consumers and the latter N-type consumers. Consumers derive utility v from consuming their preferred product, and derive a discounted utility $\tilde{k}v$ from the less preferred product, where $\tilde{k}$ is uniformly distributed over $[0, 1]$ across all consumers. Without loss of generality, we normalize v to 1. Consumers are not aware of their preferences before the search process and therefore use the search engine to explore product information. On clicking the link of a firm, consumers visit the firm's website, see the product information and the price,

[2] We focus on duopolistic analysis in the main body of the chapter. In fact, the qualitative results and main implications can be extended to oligopolistic competition. See Appendix E for details.

and learn their valuation of that firm's product. We define a consumer's net utility as the utility from consuming the product minus the price of the product. Consumers will purchase a product only when it generates a net utility exceeding a certain reservation value, which is normalized to zero. For those consumers who visit both firms' websites, they purchase from the one giving a higher (positive) net utility.

In the organic list, firms are ranked in an exogenous order reflecting their market popularity or relevance to the keyword. In this sense, representing the mainstream of the product market, firm M is listed at the i_Mth slot, which is around the very beginning of the organic list with a fairly high prominence level α_{i_M}. In contrast, firm N has a lower organic rank i_N with a lower prominence level α_{i_N} ($\alpha_{i_N} < \alpha_{i_M}$). Note that such difference in position rank and prominence level could be significant due to the existence of other non-merchant links (e.g., Wikipedia entries and news sites) appearing in between. Since the idea here is that α_{i_M} is close to 1 and α_{i_N} is significantly less than α_{i_M}, for simplification of expression, we let $\alpha_{i_M} = 1$ and $0 < \alpha_{i_N} < 1$. As we will see, the underlying spirit of this simplification is to highlight the established prominence advantage of the top organic link and the diminishing benefit of sponsored listing for the firm already occupying a prominent organic slot.

In the sponsored list, firms can bid for a prominent position. For all purposes of asymmetric duopoly analysis, let $s = 2$. The sponsored slots are sold via a second price auction, in which the firm with the highest bid wins the first sponsored slot and pays an amount equal to the second highest bid. The firm with the lower bid stays in the second sponsored slot, paying a reserve price which is normalized to zero for simplicity. Here, we ranking advertisers based on their bids on the total payment, which is in fact consistent with the common practice. In auctions of sponsored links, advertisers typically bid per-click unit prices and are ranked based on their per-click bids and the expected click-throughs on their sponsored links. Liu et al. (2010) shows that it is socially efficient to rank advertisers by the product of their per-click unit-price bids and their expected click-throughs, which exactly equals advertisers' total willingness-to-pay. In our framework, both the search engine and the advertisers rationally anticipate the expected click-throughs on the links placed at different positions. As a result, advertisers make the per-click bidding decision in the same way as if they submit a total bid. For example, if a firm in the ith organic slot and the jth sponsored slot will attract σ_{ij} sponsored clicks, to win over another bidder with $\sigma_{i'j'}$ expected sponsored clicks and per-click bid b', the firm has to bid pay-per-click b such that $b\sigma_{ij} \geq b'\sigma_{i'j'}$ (i.e., its total willingness-to-pay has to exceed its competitor's). Considering total bid rather than unit-price bid in our study is simply to avoid unnecessary assumptions on the click-through of each sponsored link, and all the analysis and results remain unaffected.

In sum, the timing of the game is as follows: In the first stage, firms submit bids for the sponsored slot. In the second stage, after observing the bidding outcome, both firms set their price simultaneously. Finally, consumers browse the SERP, sample firms' websites and make purchase decisions. Notice that consumers sample firm M's website with probability 1, regardless of the sponsored bidding outcome. Consumers sample firm N's website with probability $1-(1-\alpha_{i_N})(1-\beta_1)(1-\gamma_{i_N 1})$ if firm N wins the first sponsored slot; otherwise, they sample its website with probability $1-(1-\alpha_{i_N})(1-\beta_2)(1-\gamma_{i_N 2})$. To further simplify the notation, we define $\psi_1 \equiv (1-\alpha_{i_N})(1-\beta_1)(1-\gamma_{i_N 1})$ and $\psi_2 \equiv (1-\alpha_{i_N})(1-\beta_2)(1-\gamma_{i_N 2})$ and let $0 < \psi_1 < \psi_2 < 1$,[3] which means that winning the first sponsored slot increases the prominence level for firm N.

4.3 Equilibrium Analysis

In this section, we derive the equilibrium bidding outcome under the setting of co-listing structure that includes both organic list and sponsored links. In analyzing the equilibrium, we investigate the interplay in competing for sponsored links and provide rationale for different firms to decide their bidding strategies.

[3] $(1-\alpha_{i_N})(1-\beta_j)(1-\gamma_{i_N j}) > 0$ because $\beta_j < 1$ (due to the discounting factor of advertising) and $\gamma_{i_N j} < 1$ (due to $\gamma_{i_N j} = 1$ representing the extremely positive synergy). $(1-\alpha_{i_N})(1-\beta_j)(1-\gamma_{i_N j}) < 1$ is to exclude the unrealistic cases in which $\gamma_{i_N j}$ takes significantly negative values.

Along the line of backward induction, we start with the second stage price competition. We first formulate firms' market shares under a complete information setting, where all of the consumers are aware of the two products and know the product details and price information:

$$\begin{aligned} S_M\left(p_M, p_N\right) &= (1-\theta)\left(1-\left[p_M-p_N\right]^+\right)+\theta\left[p_N-p_M\right]^+ \\ S_N\left(p_N, p_M\right) &= \theta\left(1-\left[p_N-p_M\right]^+\right)+(1-\theta)\left[p_M-p_N\right]^+ \end{aligned} \tag{4.1}$$

where firms' prices $p_M, p_N \in [0,1]$, and $[\cdot]^+$ represents $\max\{\cdot, 0\}$. An M-type consumer will buy product M only if $1-p_M \geq \tilde{k}-p_N$, which explains the first term of $S_M\left(p_M, p_N\right)$. The other terms can be interpreted in a similar way. Notice that consumers have their own preference of one product over the other, but the degree of their preference varies, and marginal consumers exist who are almost indifferent between the two products given the price difference. Such setting allows either firm, even the niche firm, to compete for market share against its competitor, as long as it can get enough exposure.

Similarly, we can define firms' market shares in the case of informational monopoly where consumers are only aware of the one firm's product information and its price.

$$\begin{aligned} A_M\left(p_M\right) &= (1-\theta)+\theta\left(1-p_M\right) \\ A_N\left(p_N\right) &= \theta+(1-\theta)\left(1-p_N\right) \end{aligned} \tag{4.2}$$

for $p_M, p_N \in [0,1]$. In the case of A_M, for example, all M-type consumers buy from firm M, while N-type consumers buy from firm M only if $\tilde{k}-p_M \geq 0$.

Recall that because of the co-listing structure of the SERP and consumers' corresponding click behavior, consumers visit firm M's website with probability 1. Consumers visit firm N's website with probability $1-\psi_1$, if N wins the first sponsored link; otherwise, consumers visit N's website with probability $1-\psi_2$, where $0<\psi_1<\psi_2<1$. We simply denote $1-\psi$ as the probability of a consumer's visiting firm N's website and

$$\psi = \begin{cases} \psi_1 & \text{when } N \text{ wins the first sponsored slot} \\ \psi_2 & \text{otherwise} \end{cases}$$

Notice that ψ can be interpreted as a measure of information incompleteness within the market, or it can be viewed as the level of informational dominance of the mainstream firm.

A larger ψ means that the mainstream firm has greater informational advantage over the niche firm, in the sense that a larger portion of consumers is unaware of the niche firm's product. Winning the top sponsored slot can help the niche firm increase its exposure and improve the prominence level, by reducing ψ from ψ_2 to ψ_1.

Based on the notations introduced above, we can now formulate firms' demand functions for the given informational structure determined by the first stage bidding outcome (which is characterized by ψ). A proportion ψ of consumers is aware of product M only, and the other proportion is aware of both products. Therefore,

$$\begin{array}{rcl} D_M\left(p_M, p_N\right) & = & \psi A_M\left(p_M\right) + (1-\psi)\, S_M\left(p_M, p_N\right) \\ D_N\left(p_N, p_M\right) & = & (1-\psi)\, S_N\left(p_N, p_M\right) \end{array} \tag{4.3}$$

Firms' profits can thus be written as

$$\pi_i\left(p_i, p_{-i}\right) = p_i D_i\left(p_i, p_{-i}\right),\ i \in \{M, N\} \tag{4.4}$$

Based on the best responses derived from maximizing the profit function, we can derive the equilibrium prices in the second stage.

$$\begin{cases} p_M^* & = \min\{\frac{2-\theta(1-\psi)}{3(1-\theta)(1-\psi)+4\theta\psi}, 1\} \\ p_N^* & = \frac{\theta+(1-\theta)p_M^*}{2(1-\theta)} \end{cases} \tag{4.5}$$

Notice that in equilibrium, $p_N^* < p_M^*$; that is, the niche firm, which is at a disadvantage in terms of market preference, tends to cut its price to compete for market share against its stronger competitor. On the other hand, the mainstream firm tends to stay away from the intense price competition when it has significant informational advantage. In fact, $p_M^* = 1$ when $\psi \geq \frac{1}{3}$, according to Eq.(4.5). As long as its informational advantage is reasonably large, firm M forgoes the price competition with N and simply charges the monopoly price to fully exploit its guaranteed demand.

Next, we derive firms' equilibrium profits from the second stage price competition, $\pi_i^*\left(\psi, \theta\right)$, as a function of the information incompleteness level ψ and the market asymmetry level θ. By substituting $\{p_M^*, p_N^*\}$ into Eq.(4.4), we summarize $\pi_i^*\left(\psi, \theta\right)$ in Table 4.1. Lemma 4.1 describes an important property of the equilibrium profit functions.

Table 4.1: Equilibrium Profit Functions in the Second Stage Price Competition

	$0 < \psi < \frac{1}{3}$	$\frac{1}{3} \le \psi < 1$
$\pi_M^* (\psi, \theta)$	$\frac{[2-\theta(1-\psi)]^2[(1-\theta)-(1-2\theta)\psi]}{[3(1-\theta)(1-\psi)+4\theta\psi]^2}$	$\left(\frac{1}{2}-\theta\right)\psi+\frac{1}{2}$
$\pi_N^* (\psi, \theta)$	$\frac{[(1+\theta)(1-\theta)+\theta(3\theta-1)\psi]^2(1-\psi)}{(1-\theta)[3(1-\theta)(1-\psi)+4\theta\psi]^2}$	$\frac{1-\psi}{4(1-\theta)}$

Lemma 4.1. *Both firms' equilibrium profit functions are submodular, that is,* $\frac{\partial^2}{\partial\psi\partial\theta}\pi_i^* (\psi, \theta) < 0,\ i \in \{M, N\}$.

Lemma 4.1 reveals the trend that the marginal benefit of winning a better sponsored slot ($\frac{\partial}{\partial\psi}\pi_i^* (\psi, \theta)$) changes with the market structure (θ). It is a crucial step toward the revelation of the bidding outcome , as we will discuss in detail soon. As we can show, $\frac{\partial}{\partial\psi}\pi_M^* (\psi, \theta) > 0$, indicating that firm M benefits from enlarging its informational dominance by keeping its competitor's prominence level low. Thus, the submodularity of $\pi_M^* (\psi, \theta)$ implies that such marginal benefit *decreases* as θ increases, or, in other words, as the two firms become more comparable. Likewise, generically, $\frac{\partial}{\partial\psi}\pi_N^* (\psi, \theta) < 0$, meaning that firm N has incentive to reduce ψ by winning a prominent sponsored slot, and the submodularity of $\pi_N^* (\psi, \theta)$ implies that such incentive *increases* as θ increases.

Based on the equilibrium profits in the second stage price competition, we next investigate the bidding competition in the first stage. In second-price auctions, it is well documented that bidding true value is a weakly dominant strategy for all bidders. Therefore, in the first stage, the unique perfect equilibrium is that both firms bid their true value b_i^*:

$$\begin{cases} b_M^* &= [\pi_M^* (\psi_2, \theta) - \pi_M^* (\psi_1, \theta)]^+ \\ b_N^* &= [\pi_N^* (\psi_1, \theta) - \pi_N^* (\psi_2, \theta)]^+ \end{cases} \tag{4.6}$$

which equals their respective equilibrium profit difference between winning the first sponsored slot and otherwise, bounded below at zero.

Applying the results from Lemma 4.1, we uncover the bidding outcome as follows.

Proposition 4.1. *In equilibrium, there exists a cutoff* $\theta^*(\psi_1, \psi_2)$*, such that when* $0 < \theta < \theta^*(\psi_1, \psi_2)$*,* M *bids higher and wins the first sponsored slot, and when* $\theta^*(\psi_1, \psi_2) < \theta < \frac{1}{2}$*,* N *outbids its rival. Here,* $\theta^*(\psi_1, \psi_2) \in (0, \frac{1}{2})$ *and is defined by*

$$\pi_M^*(\psi_2, \theta^*) - \pi_M^*(\psi_1, \theta^*) = \pi_N^*(\psi_1, \theta^*) - \pi_N^*(\psi_2, \theta^*) \tag{4.7}$$

Now we are able to provide reasonable answers to some of our initial research questions: What is a firm's incentive to bid for a prominent sponsored slot when it is already placed at a prominent position in the organic list? How does organic listing affect firms' bidding for sponsored links? How does the effect differ across differentiated firms?

Under the structure that organic list is paralleled with sponsored list, a prominent sponsored link benefits its winner in at least two aspects. The first aspect is the *promotive effect*. Winning a prominent sponsored slot increases a firm's probability of being noticed via the additional sponsored click-throughs, and it may also create significant synergy between the organic and sponsored lists to further enhance the exposure for the firm. The second aspect can be viewed as the *preventive effect*. The firm that wins the prominent sponsored slot can keep its competitor away from that position and effectively prevent the competitor from increasing its exposure, and can thus reap its informational advantage.

As prominent organic links usually capture the most attention within a SERP, there is little room to improve exposure for those firms with top organic ranks. In this sense, those firms' incentive of bidding for sponsored slots mainly originates from the preventive side. In contrast, a firm with a less prominent organic rank finds motive more from the promotive rather than the preventive perspective, as winning a prominent sponsored link greatly complements its inadequate attention level from the organic list but can barely affect the high click-throughs attained by its competitors via their prominent organic positions. Therefore, as a firm's organic rank improves, the promotive effect of the sponsored listing for the firm decreases and the preventive effect increases. We capture this trend in the inherent model setup, and further highlight the trend by letting $\alpha_{i_M} = 1$ such that the top sponsored link has no promotive effect for firm M and no preventive effect for firm N accordingly. As mentioned before, this setup allows us to disentangle the two otherwise intertwined effects, tease out the interplay between the two effects in the bidding competition, and deliver neat insight on how such interaction evolves with the market structure.

In this framework, we can think of $\frac{\partial}{\partial \psi}\pi_M^*(\psi, \theta)$ as a measure of the marginal preventive effect of the sponsored listing, and consider $-\frac{\partial}{\partial \psi}\pi_N^*(\psi, \theta)$ measuring the marginal promotive effect. Lemma 4.1 shows the dynamic evolving of the two interacting effects when the market structure changes. As the market asymmetry decreases (i.e., θ increases), the marginal promotive effect of the sponsored listing increases and the marginal preventive effect decreases. This is because when the market preference becomes more diverse, the niche firm is more comparable to its competitor, and it can thus considerably improve its profit by increasing its prominence level via winning the prominent sponsored slot. In contrast, the market share that the mainstream firm can capture becomes less, even under informational monopoly, so the mainstream firm benefits less from blocking its competitor. Since the two effects evolve in the opposite directions as the market structure changes, naturally, there exists a threshold θ^*, a certain cutoff level of market asymmetry, such that the promotive effect dominates when θ is above that threshold and the preventive effect dominates when θ is below that threshold, as is summarized by Proposition 4.1. Eq. (4.7) states that at the cutoff level of market asymmetry, the promotive benefit for N equals the preventive benefit for M.

Proposition 4.1 also provides rationale for firms in different types of markets to determine the value of a prominent sponsored slot in bidding competition. In a highly asymmetric market, the leading firm should bid aggressively to win the top sponsored slot because occupying a prominent position in both the organic and sponsored list enlarges its informational dominance, ensures its advantageous position in price competition, and greatly improves its sales profit. However, when the market preference is relatively diversified, the firm at a disadvantage in terms of market preference should bid to win a prominent sponsored slot, especially when it is placed at a lower organic position with an unsatisfactory attention level. This is because its marginal benefit from improving its prominence level is fairly high in this case.

Notice that the niche firm's incentive to win the top sponsored link $(-\frac{\partial}{\partial\psi}\pi_N^*(\psi,\theta))$ decreases as θ goes down. It is thus worth pointing out that in a certain parameter region, such incentive can fall so low that firm N is not willing to bid any positive amount.

Corollary 4.1. *The niche firm may have no incentive for sponsored bidding; in particular, when $\psi_2 < \frac{1}{3}$ and $\theta < \frac{17-\sqrt{145}}{24}$, $b_N^* = 0$.*

When the market preference is highly asymmetric (i.e., θ is small), if the niche firm can gain a reasonable level of attention from the organic link and the second sponsored link (i.e., $\psi_2 < \frac{1}{3}$), it will not bother to bid for the top sponsored link at all. The reason is that should N win the first sponsored slot and increase its prominence level to $1 - \psi_1$, it would trigger an intense price war with firm M, which eventually results in an even lower equilibrium profit level for firm N. Weak in competence, the niche firm is better off staying in a relatively low prominence level and therefore has no intention to bid for the top sponsored link at all. The driving force here is the unshakable prominence advantage given to the leading firm in the organic list.

4.4 Effects of Organic Listing

Having derived the equilibrium under the co-listing structure, in this section, we further investigate the effects of organic listing on the equilibrium outcomes. We first construct a case absent of organic listing as a benchmark, and then compare the equilibrium outcomes under the benchmark case with that derived from Section 4.3 in three aspects: overall social welfare, resulting sales diversity, and search engine benefit.

4.4.1 A Benchmark Case

As a benchmark, we consider a case where each SERP contains the sponsored list only. The benchmark case can be imagined as the search engine's choosing to display only one list of links, all of which are potential advertising slots to be sold. Similar practices can be found in some regional search engines, such as Baidu.com, the leading search engine in China, and in early versions of search advertising, such as those used by Goto.com and, later, Overture.com, in which there was only one list, mixed with paid advertising links and organic search results, and advertisers could bid for their ranks. We call the original case with both organic list and sponsored list the co-listing case.

To be consistent with the original model, we consider two sponsored links on a SERP. An individual consumer clicks the first sponsored link with probability q_1 and the second with probability q_2 ($q_1 > q_2$). To make the benchmark case comparable to the co-listing case, we let $q_1 = 1$ to model the dominant prominence of the first link on a webpage because it can capture the most user attention, just as the top organic link does in the co-listing case. Similarly to the co-listing case, we denote $q_2 \equiv 1 - \psi$, where ψ measures the level of information incompleteness or informational dominance. All other settings (i.e., firms, market preference, auction rules) follow the original model in Section 4.2.

Similarly, we start the analysis from the second stage price competition. We can formulate firms' demand functions when firm i wins the first sponsored slot while the other firm $\bar{i}$ stays in the second, $\{i, \bar{i}\} = \{M, N\}$:

$$\begin{aligned} D_i\left(p_i, p_{\bar{i}}\right) &= \psi A_i\left(p_i\right) + (1-\psi)\, S_i\left(p_i, p_{\bar{i}}\right) \\ D_{\bar{i}}\left(p_{\bar{i}}, p_i\right) &= (1-\psi)\, S_{\bar{i}}\left(p_{\bar{i}}, p_i\right) \end{aligned} \tag{4.8}$$

where p_i and $p_{\bar{i}}$ are the firms' prices, and $A_i(\cdot)$ and $S_i(\cdot,\cdot)$ are defined by Eq.(4.2) and Eq.(4.1), respectively. When firm M wins the first sponsored slot and thus attracts most of the attention, as in the co-listing case, the demand function is exactly the same as before (see Eq.(4.3)). Therefore, both firms face the same competitive situation, and the equilibrium prices and profits remain in the same format as Eq.(4.5). The main difference, however, arises when firm N wins the first sponsored slot and firm M can only have the less-prominent sponsored position. Notice that the mainstream firm now could become informationally disadvantaged compared to its competitor, because it no longer has a guaranteed prominence dominance from occupying the top organic link as in the co-listing

Table 4.2: Equilibrium Profits in the Second Stage Price Competition (Benchmark Case)

	When M wins		When N wins
	$0 < \psi < \frac{1}{3}$	$\frac{1}{3} \leq \psi < 1$	
M's profit	$\hat{\pi}_M^1 = \frac{[2-\theta(1-\psi)]^2[(1-\theta)-(1-2\theta)\psi]}{[3(1-\theta)(1-\psi)+4\theta\psi]^2}$	$\hat{\pi}_M^1 = \left(\frac{1}{2}-\theta\right)\psi + \frac{1}{2}$	$\hat{\pi}_M^2 = \frac{(1-\psi)(2+\psi-\theta-\theta\psi)^2}{(3+\psi)^2(1-\theta)}$
N's profit	$\hat{\pi}_N^2 = \frac{[(1+\theta)(1-\theta)+\theta(3\theta-1)\psi]^2(1-\psi)}{(1-\theta)[3(1-\theta)(1-\psi)+4\theta\psi]^2}$	$\hat{\pi}_N^2 = \frac{1-\psi}{4(1-\theta)}$	$\hat{\pi}_N^1 = \frac{(1+\psi+\theta-\theta\psi)^2}{(3+\psi)^2(1-\theta)}$

case. In other words, the niche firm can now win over significant prominence advantage to better compete for market shares. We derive the equilibrium prices when N wins the first sponsored position as follows.

$$\begin{cases} \hat{p}_M = \frac{2+\psi-\theta-\theta\psi}{(3+\psi)(1-\theta)} \\ \hat{p}_N = \frac{1+\psi+\theta-\theta\psi}{(3+\psi)(1-\theta)} \end{cases} \tag{4.9}$$

The equilibrium profit from the second stage price competition can be derived in a similar way, as summarized by Table 4.2, where $\hat{\pi}_i^j$ is firm i's equilibrium profit in the jth sponsored position in the benchmark case ($i \in \{M, N\}$, $j \in \{1, 2\}$).

In the first stage bidding competition, both firms bid their true value: $\hat{b}_i = [\hat{\pi}_i^1 - \hat{\pi}_i^2]^+$ ($i \in \{M, N\}$), which again is the difference between the equilibrium profits when they win the first sponsored slot and when they do not. By comparing two firms' equilibrium bids $\hat{b}_M$ and $\hat{b}_N$, we can uncover the bidding outcome in the benchmark case.

Proposition 4.2. *In the benchmark case, in equilibrium, M always bids higher and wins the first sponsored slot; that is, $\hat{b}_M > \hat{b}_N$.*

Proposition 4.2 shows that the only possible outcome of the bidding competition is that the firm with advantage in market preference wins the most prominent sponsored slot. Surprising as it might seem, this result can be well understood within the framework of the two aforementioned interacting effects. Recall that in the co-listing case, a prominent sponsored link engenders mainly preventive effect for the mainstream firm and mainly promotive effect for the niche firm. In contrast, in the benchmark case, winning a prominent sponsored slot engenders both promotive and preventive effects for either of the two firms. These two effects are amplified when a firm has greater competence in terms of market preference. As a result, as long as firm M has market preference advantage over N (i.e., $\theta < \frac{1}{2}$), firm M always has greater incentive to win the top sponsored slot compared to firm N, regardless of the preference advantage magnitude.

It is also worth noting that in equilibrium both firms adopt the same pricing strategies as in the co-listing case (according to Eq.(4.5)) and firm M charges a higher price than firm N.

Corollary 4.2. *In the benchmark case, the niche firm has a positive bidding incentive in most cases; that is, when* $\frac{5\sqrt{6}}{6} - 2 < \theta < \frac{1}{2}$, $\hat{b}_N > 0$.

Compared to Corollary 4.1, in the benchmark case, the niche firm bids a positive amount in a larger region of parameter value. This result also implies that firms' bidding incentive could be greater in the benchmark case than in the co-listing case.

Next, we compare the equilibrium outcome in the co-listing case with that in the benchmark case and investigate the effects of organic listing on social welfare, sales diversity, and the search engine's benefit. To focus on systematic difference rather than perplexing discussions on parameter values, we let $\psi = \psi_2$ so that we focus on a typical case in which firm N receives a similar level of exposure in both cases when it does not win the prominent sponsored slot. The parametric assumption is mainly to facilitate a neat comparison and does not affect our qualitative results. We use superscript C to denote the co-listing case and B to denote the benchmark case.

4.4.2 Effect on Social Welfare

We consider the social welfare as the sum of total consumer surplus, both firms' profits, and the search engine's revenue. Essentially, social welfare equals the sum of the realized utility of consumers from consuming the products they have purchased. Recall that M always outbids N in equilibrium in the benchmark case and M's equilibrium price is higher than N's in both cases. We therefore can write the equilibrium social welfare as a function of information incompleteness ψ in a uniform expression.

$$\begin{aligned} W(\psi,\theta) = \ & \psi\left[(1-\theta)+\theta\int_{p_M^*(\psi,\theta)}^{1} x dx\right] + (1-\psi)(1-\theta)\left(1-p_M^*(\psi,\theta)+p_N^*(\psi,\theta)\right) \\ & +(1-\psi)\left[\theta+(1-\theta)\int_{1-p_M^*(\psi,\theta)+p_N^*(\psi,\theta)}^{1} x dx\right] \end{aligned} \tag{4.10}$$

where $p_M^*(\psi,\theta)$ and $p_N^*(\psi,\theta)$ are defined by Eq.(4.5). The first term represents the total realized utility of those consumers who visit firm M's website only and make purchases there. The second term refers to those who visit both firms' sites and buy from firm M, and the third term refers to those who buy from N after visiting both sites. By definition, the equilibrium social welfare in the benchmark case $W^B = W(\psi_2,\theta)$. The equilibrium welfare in the co-listing case depends on the bidding outcome according to Proposition 4.1: $W^C = W(\psi_2,\theta)$ when $\theta < \theta^*(\psi_1,\psi_2)$, and $W^C = W(\psi_1,\theta)$ when $\theta > \theta^*(\psi_1,\psi_2)$. As we can see, when M wins the top sponsored slot, the welfare achieved in both cases is the same. What we are interested in is whether the presence of the organic list can increase total social welfare when the niche firm wins the top sponsored slot. The answer is positive.

Proposition 4.3. *Organic listing improves the social welfare in that $W^C \geq W^B$ and the strict inequality holds when $\theta > \theta^*(\psi_1,\psi_2)$* (defined by Eq.(4.7)).

When $\theta > \theta^*(\psi_1,\psi_2)$, N wins the prominent sponsored slot in the co-listing case and increases its exposure rate from $1-\psi_2$ to $1-\psi_1$. Proposition 4.3 reveals that increasing the niche firm's exposure improves the total welfare. Notice that social welfare achieves its maximum when all consumers purchase their preferred products. From Eq.(4.10), we can identify two main sources of social efficiency loss: One is informational incompleteness (characterized by the first term in Eq. (4.10)) and the other is lack of competitiveness (characterized by the second and third terms). When ψ is high, the *informational* efficiency loss is high in the sense that many N-type consumers are not aware of product N and end up buying from M instead. Furthermore, a portion of N-type consumers who visit only

M's website may leave with no purchase because of the high price charged by firm M. There is also *competitive* efficiency loss: When the degree of its informational advantage is high, firm M tends to charge a high price. Consequently, a certain portion of marginal M-type consumers who visit both sites may purchase product N rather than product M, which is socially inefficient. When ψ is reduced, N increases its exposure and M charges a more competitive price in equilibrium, so both informational and competitive efficiency losses are mitigated, which leads to an increase in the overall social welfare.

Different from the benchmark case, in the co-listing case, the free prominence advantage given to the leading firm in the organic list diminishes its sponsored bidding incentive because of the decreased promotive benefit. It thus helps the niche firm to win the prominent sponsored link and better expose itself. In this sense, organic listing adjusts the competence difference among firms by compensating the dominant firm and thus balances the equilibrium informational structure to improve overall social welfare.

4.4.3 Effect on Sales Diversity

We use the *Gini coefficient* to measure the sales diversity. As a popular measure of inequality of income distribution, the Gini coefficient is defined as $G = 1 - 2\int_0^1 L(x)\,dx$, where $L(x)$ is the Lorenz curve, which measures the lowest $100x$ percent population's cumulative income percentage. The Gini coefficient measures the difference between the actual (income) distribution and the perfect equality/diversification case. A higher Gini coefficient indicates a greater degree of inequality, while a lower coefficient means greater diversification. For discrete cases, the Gini coefficient can be computed as follows,

$$G = 1 - \frac{\frac{1}{n}\sum_{i=1}^{n}(S_{i-1} + S_i)}{S_n}, \tag{4.11}$$

where $S_i = \sum_{j=1}^{i} y_j$ (and $S_0 \equiv 0$), and $\{y_i\}_{i=1}^{n}$ is the ordered sequence of the value of interest (e.g., income) for each individual in the population such that $y_i \leq y_{i+1}$.

In our model, we use the Gini coefficient to measure the diversification of the realized sales across the two firms in equilibrium, and the value of interest in Eq.(4.11) is the equilibrium sales amount. By substituting the equilibrium sales amount $D_N\left(p_M^*\left(\psi\right), p_N^*\left(\psi\right);\psi\right)$ and $D_M\left(p_M^*\left(\psi\right), p_N^*\left(\psi\right);\psi\right)$ derived from Eq.(4.3) into Eq.(4.11), we can calculate the Gini coefficient here as follows.

$$G\left(\psi\right) = \frac{1}{2} - \frac{D_N\left(p_M^*\left(\psi\right), p_N^*\left(\psi\right);\psi\right)}{D_N\left(p_M^*\left(\psi\right), p_N^*\left(\psi\right);\psi\right) + D_M\left(p_M^*\left(\psi\right), p_N^*\left(\psi\right);\psi\right)} \tag{4.12}$$

Similarly as before, we have $G^B = G\left(\psi_2\right)$; $G^C = G\left(\psi_2\right)$ when $\theta < \theta^*\left(\psi_1, \psi_2\right)$ and $G^C = G\left(\psi_1\right)$ when $\theta > \theta^*\left(\psi_1, \psi_2\right)$.

Proposition 4.4. *Organic listing improves the sales diversity in that $G^C \leq G^B$ and the strict inequality holds when $\theta > \theta^*\left(\psi_1, \psi_2\right)$* (defined by Eq.(4.7)).

Similar to the welfare-improving effect, organic listing increases sales diversity when the niche firm outbids the mainstream one. By winning the prominent sponsored position under the co-listing structure, the niche firm increases its exposure and attracts more consumers to visit its site, among whom all the N-type consumers as well as part of the M-type consumers purchase from it. Therefore, the realized market share of the niche firm is increased and the overall sales diversity is improved. In this sense, the diversity-improving effect of organic listing and the aforementioned welfare-improving effect share the same origin: adjusting the competence difference among firms by compensating the strong.

4.4.4 Effect on Search Engine Benefit

In evaluating the search engine's benefit, we consider both the short-term and the long-term aspects. We decompose the search engine benefit, SB, into two parts: immediate revenue, IR, and long-term growth, LG. Because we are interested in the general trends rather than the detailed dynamics, we model search engine benefit in a general and abstract way such that SB is defined as a function of IR and LG, $SB = f\left(IR, LG\right)$, where $f\left(\cdot,\cdot\right)$ is strictly increasing in both dimensions.

IR is the search engine's revenue from sponsored bidding when holding the total consumer base constant (and normalized to 1), and LG reflects the change of future customer base. We consider LG as a strictly increasing function of the equilibrium consumer surplus, CS, so that $LG = g(CS)$. Here, CS is defined as follows.

$$\begin{aligned} CS(\psi,\theta) = \;& \psi\left[(1-\theta)(1-p_M^*(\psi,\theta)) + \theta\int_{p_M^*(\psi,\theta)}^{1}(x-p_M^*(\psi,\theta))\,dx\right] \\ & +(1-\psi)(1-\theta)(1-p_M^*(\psi,\theta)+p_N^*(\psi,\theta))(1-p_M^*(\psi,\theta)) \\ & +(1-\psi)\left[\theta(1-p_N^*(\psi,\theta)) + (1-\theta)\int_{1-p_M^*(\psi,\theta)+p_N^*(\psi,\theta)}^{1}(x-p_N^*(\psi,\theta))\,dx\right], \end{aligned} \tag{4.13}$$

where $p_M^*(\psi,\theta)$ and $p_N^*(\psi,\theta)$ are equilibrium prices as before. Notice that Eq.(4.13) equals the expected net utility of any individual customer before entering the search market (without knowing her actual preference before the search). In this sense, we can interpret $g(CS)$ as the future consumer traffic volume attracted to the search engine when consumers have outside options (e.g., competing search engines). More specifically, we may consider that consumers have different reserve utilities which follow a certain distribution, and they use this particular search engine if the expected utility exceeds their reserve values and leave for other options otherwise. As a result, the future consumer traffic volume forms an increasing function of CS, as can be represented by $g(CS)$.

In sum, search engine benefit is modeled as

$$SB = f(IR, LG) = f(IR, g(CS)) = h(IR, CS), \tag{4.14}$$

where $\frac{\partial}{\partial IR}h > 0$ and $\frac{\partial}{\partial CS}h > 0$. Notice that the general form of the search engine's benefit leaves sufficient flexibility so that SB can either be interpreted as the net present value of all future revenue flows in a dynamic context or be considered as incorporating benefits from other non-monetary factors (e.g., reputation and public image).

We are interested in how organic listing affects the search engine's benefit in both short-term and long-term perspectives. Thus, we next compare the immediate revenue and the long-term growth (i.e., consumer surplus, essentially) under both the benchmark and the co-listing cases.

In second-price auctions, the auctioneer's revenue equals the second highest bid in equilibrium. In the benchmark case, since M always wins the auction, search engine's immediate revenue $IR^B = \hat{b}_N$. In the case with the organic list, $IR^C = b_N^*$ when $\theta < \theta^*(\psi_1, \psi_2)$ and $IR^C = b_M^*$ when $\theta > \theta^*(\psi_1, \psi_2)$, where $\theta^*(\psi_1, \psi_2)$ is defined by Eq.(4.7).

Lemma 4.2. *Generically, the search engine's immediate revenue is lower in the co-listing case; that is, when* $\frac{5\sqrt{6}}{6} - 2 < \theta < \frac{1}{2}$, $IR^C < IR^B$ *for all* $0 < \psi_1 < \psi_2 < 1$.

Lemma 4.2 shows that compared to the case without the organic list, the co-listing case may reduce the search engine's revenue in the short run. In the spirit of Corollary 4.1 and Corollary 4.2, this somewhat disappointing result should not sound too surprising. It underscores the fact that the free prominence advantage given to the leading firm in the organic list may reduce advertisers' bidding incentives and hence sacrifice the search engine's direct revenue.

A brief reasoning for the above result is as follows. Since $IR^B = \hat{b}_N$ and $IR^C = \min\{b_M^*, b_N^*\}$, to conclude that $IR^B > IR^C$, it is sufficient to show that $\hat{b}_N > b_N^*$. Recall that a firm's equilibrium bid equals the difference between its equilibrium profits when winning the top sponsored slot and when not winning it (i.e., according to Tables 4.1 and 4.2, $b_N^* = \pi_N^*(\psi_1, \theta) - \pi_N^*(\psi_2, \theta)$ and $\hat{b}_N = \hat{\pi}_N^1 - \hat{\pi}_N^2$). On the one hand, as discussed earlier, when N does not win the top sponsored slot, the equilibrium profit achieved in both cases is the same (i.e., $\pi_N^*(\psi_2, \theta) = \hat{\pi}_N^2$). On the other hand, when N wins the top sponsored slot, in the co-listing case, M still possesses significant prominence from the organic list, which limits N's profit due to the consequent intense price competition; in the benchmark case, however, N could overturn the informational dominance structure thoroughly and greatly improve the profitability of winning the top sponsored slot. As a result, N's winning profit in the benchmark case is higher in general (i.e., $\hat{\pi}_N^1 > \pi_N^*(\psi_1, \theta)$). In consequence, as long as $\hat{b}_N > 0$ (i.e., under some boundary condition on θ according to Corollary 4.2), we can conclude that $\hat{b}_N > b_N^*$ and hence $IR^B > IR^C$.

It is also worth noting that, in addition to N's decreased bidding incentive, firm M's incentive to bid for a sponsored link is also less under the co-listing case than under the benchmark case, sometimes even to a greater degree such that firm M might lose the bidding competition to firm N in the co-listing case. In other words, paralleling the organic list with the sponsored list generally results in lower bidding incentives for both firms.

The effect on the equilibrium consumer surplus can be analyzed in a similar fashion as the social welfare and the sales diversity. By Eq.(4.13), $CS^B = CS(\psi_2, \theta)$ and $CS^C = CS(\psi_1, \theta)$ when $\theta > \theta^*(\psi_1, \psi_2)$.

Lemma 4.3. *The equilibrium consumer surplus in the co-listing case is higher in that $CS^C \geq CS^B$ and the strict inequality holds when $\theta > \theta^*(\psi_1, \psi_2)$* (defined by Eq.(4.7)).

The above result can be understood based on the same two sources of loss in analyzing the equilibrium social welfare: informational and competitive. In the co-listing case, the diluted bidding incentive of the leading firm gives the niche firm more chances to win a better sponsored position for a better exposure. As a result, consumers are more likely to find the product they prefer—at lower prices due to the intensified price competition. Therefore, by inducing a lower level of informational incompleteness ψ in equilibrium, the co-listing case improves the overall consumer surplus.

It is worth noting that the co-listing structure manages to induce the niche firm to increase its exposure in the sponsored list while keeping the organic list "organic." Therefore, it increases the surplus of the potential consumers (i.e., those who are looking for product information) and meanwhile ensures the general search engine users' utilities. Such effects promise growth of the user base in the long run.

Combining the results from Lemma 4.2 and Lemma 4.3, we can conclude that the effect of organic listing on the search engine's benefit is a balance between short-term profitability and long-term growth.

Proposition 4.5. *Organic listing trades off the short-term revenue for the long-term benefit of the search engine in that $IR^C < IR^B$ and $LG^C > LG^B$ when $\theta > \max\{\theta^*(\psi_1, \psi_2), \frac{5\sqrt{6}}{6} - 2\}$.*

An immediate result from Proposition 4.5 is that if the search engine values the long-term benefit enough (i.e., if $\frac{\partial}{\partial LG} f \gg \frac{\partial}{\partial IR} f$ as in Eq.(4.14)), then the overall search engine benefit can be much higher in the co-listing case. In this sense, separating the organic list out as an independent major list is essentially a choice between myopic and long-sighted perspectives.

To conclude, this section analyzes the effects of organic listing on the equilibrium outcomes. Overall, organic listing reduces search engine revenue in the short run but improves the equilibrium social welfare, sales diversity, and consumer surplus, which eventually benefits the search engine in the long run.

It is worth pointing out that the aforementioned beneficial effects of organic listing function only under moderate levels of market asymmetry (i.e., firms are not too different in market preference so θ is not too small), which can be considered as the general cases for most markets in reality. It hence justifies the design of organic listing in general. Nevertheless, these effects malfunction when the competence difference among advertisers is too large to adjust. For firms at the very tail end of the distribution (i.e., for niche firms with very small θ), their bidding incentive is still lower than that of their strong competitors. As a result, the leading firms dominate the prominence in both lists and no fundamental change occurs in information structure after the sponsored bidding competition. Consequently, while bearing revenue loss, the search engine fails to engender structural improvement in social welfare, sales diversity, as well as consumer surplus. In this sense, the typical organic listing design serves the "middles" well, rather than the "tails."

4.5 Improving Organic Ranking

The previous section discusses the beneficial effects of organic listing and also pinpoints the malfunction of such effects with regard to "tail" firms. In this section, we propose a possible direction that could mitigate such drawbacks and can better serve the tails.

Instead of assuming that the organic list ranks firms in an order based strictly on their relative popularity (which we refer to as *pure* organic ranking mechanism), we consider a *mixed* organic ranking mechanism in which the organic list ranks firms by their popularity with certain probability and in an *inverse* order otherwise. Following the model setup in Section 4.2, a mixed organic ranking mechanism can be characterized by a randomization parameter λ ($0 \leq \lambda \leq 1$) such that with probability $1-\lambda$ the mainstream firm is listed in a prominent organic slot i_M on the SERP and the niche firm is listed in a less-prominent organic slot i_N; meanwhile, with probability λ, firm N is given a prominent organic position i'_N and M stays in a less-prominent organic position i'_M. Notice that $\alpha_{i_M} > \alpha_{i_N}$, $\alpha_{i'_N} > \alpha_{i'_M}$, and the pure organic ranking is thus a special case of mixed ranking with $\lambda = 0$.

To be consistent with the baseline model, we let $\alpha_{i'_N} = \alpha_{i_M} = 1$ to model the significant attention-catching effect of a prominent organic position. When firm M wins the first sponsored slot, the probability of M's being visited by a consumer (q^1_M) and the probability of N's being visited (q^2_N) are

$$\begin{aligned} q^1_M &= (1-\lambda) + \lambda\left[1-(1-\alpha_{i'_M})(1-\beta_1)(1-\gamma_{i'_M 1})\right] \\ q^2_N &= (1-\lambda)\left[1-(1-\alpha_{i_N})(1-\beta_2)(1-\gamma_{i_N 2})\right] + \lambda \end{aligned} \tag{4.15}$$

Similarly, when firm N wins the first sponsored slot, the probability of N's being visited (q^1_N) and that of M's being visited (q^2_M) are

$$\begin{aligned} q^2_M &= (1-\lambda) + \lambda\left[1-(1-\alpha_{i'_M})(1-\beta_2)(1-\gamma_{i'_M 2})\right] \\ q^1_N &= (1-\lambda)\left[1-(1-\alpha_{i_N})(1-\beta_1)(1-\gamma_{i_N 1})\right] + \lambda \end{aligned} \tag{4.16}$$

To simplify the discussion, we let $(1-\alpha_{i'_M})(1-\beta_1)(1-\gamma_{i'_M 1}) = (1-\alpha_{i_N})(1-\beta_1)(1-\gamma_{i_N 1}) \equiv \psi_1$ and $(1-\alpha_{i_N})(1-\beta_2)(1-\gamma_{i_N 2}) = (1-\alpha_{i'_M})(1-\beta_2)(1-\gamma_{i'_M 2}) \equiv \psi_2$, $\psi_1 < \psi_2$. A mixed organic ranking mechanism that randomly switches M's and N's positions with probability λ is an example satisfying the above equality. For simplicity, here we let $\psi_1 = 0$ because the first sponsored link often attracts significant attention so that $(1-\beta_1)$ can be sufficiently small. The parametric assumptions are only to facilitate derivation of neat analytical results and are not essential for the qualitative results to hold. As we can show numerically, insights remain the same when ψ_1 is extended to be positive.

The underlying idea of the mixed organic ranking mechanism is to strategically introduce uncertainty in firms' organic ranks so that firms' relative prominence advantage can be reversed probabilistically. In practice, mixed ranking can be implemented by simply randomizing the ordering of targeted advertisers with the desired probability. It can also be interpreted as including additional factors other than the popularity measure into the organic ranking rule.

We can formulate firms' demand functions in a similar way. When firm M wins the first sponsored slot, M possesses informational monopoly power if firm N is placed in the less prominent organic slot (with probability $1-\lambda$) and is not noticed by a consumer (with probability ψ_2). Therefore, the demand functions facing both firms are as follows.

$$\begin{array}{rcl} D_M^1\left(p_M, p_N\right) & = & \left(1-\lambda\right)\psi_2 A_M\left(p_M\right) + \left[1-\left(1-\lambda\right)\psi_2\right] S_M\left(p_M, p_N\right) \\ D_N^2\left(p_N, p_M\right) & = & \left[1-\left(1-\lambda\right)\psi_2\right] S_N\left(p_N, p_M\right) \end{array} \tag{4.17}$$

where $A_i\left(\cdot\right)$ and $S_i\left(\cdot\right)$ are defined as before by Eq.(4.1) and Eq.(4.2). When firm N wins the sponsored bidding, similarly, the demand functions become

$$\begin{array}{rcl} D_M^2\left(p_M, p_N\right) & = & \left(1-\lambda\psi_2\right) S_M\left(p_M, p_N\right) \\ D_N^1\left(p_N, p_M\right) & = & \lambda\psi_2 A_N\left(p_N\right) + \left(1-\lambda\psi_2\right) S_N\left(p_N, p_M\right) \end{array} \tag{4.18}$$

Comparing the above with Eq.(4.3), we can see the major difference from the original model with pure organic ranking. By strategically randomizing the organic ranking and probabilistically altering the relative prominence advantage in the organic list, the mainstream firm no longer possesses guaranteed prominence advantage. In particular, if it fails to win the sponsored bidding, the mainstream firm might even become inferior to its competitor in terms of informational exposure. Hence, it is now possible for the niche firm to gain a certain level of prominence dominance by winning the prominent sponsored position.

Along a similar approach as before, by considering the best response to its competitor's price in the second stage, we can first derive firms' equilibrium price conditional on whether they acquire the first or the second sponsored position, $\tilde{p}_i^j$, where

$\{i, \bar{i}\} = \{M, N\}$ and $\{j, \bar{j}\} = \{1, 2\}$. In other words, $\tilde{p}_i^j$ solves the profit maximization problem $\max_p pD_i^j\left(p, \tilde{p}_{\bar{i}}^{\bar{j}}\right)$. Plugging $\{\tilde{p}_i^j, \tilde{p}_{\bar{i}}^{\bar{j}}\}$ back into the profit function, we can derive the second-stage equilibrium profits for firms in both cases: $\tilde{\pi}_i^j = \tilde{p}_i^j D_i^j\left(\tilde{p}_i^j, \tilde{p}_{\bar{i}}^{\bar{j}}\right)$. Back to the first-stage bidding competition, again, bidding the true value $\Delta\tilde{\pi}_i = \tilde{\pi}_i^1 - \tilde{\pi}_i^2$ is the unique perfect equilibrium, that is, $\tilde{b}_i = [\Delta\tilde{\pi}_i]^+$, $i \in \{M, N\}$.

We are interested to see whether introducing mixed organic ranking can improve the equilibrium outcome, especially when θ is small. In particular, we investigate the marginal effect of increasing the randomizing parameter λ on the equilibrium welfare, sales diversity, and both the short-term and the long-term benefit of the search engine, evaluated at $\lambda = 0$ (which is the case of pure organic ranking). Notice that when $\lambda = 0$, the equilibrium outcome is the same as in the original model: When $\theta < \theta^*(0, \psi_2)$ (defined by Eq.(4.7)), we have $\tilde{b}_M > \tilde{b}_N$ and $\tilde{p}_M^1 > \tilde{p}_N^2$. As a result, the equilibrium revenue for the search engine equals $\tilde{IR}(\psi_2, \theta, \lambda) = [\Delta\tilde{\pi}_N(\psi_2, \theta, \lambda)]^+$. The equilibrium consumer surplus and social welfare can be written as

$$\begin{aligned} \tilde{CS}(\psi_2, \theta, \lambda) = \;& (1-\lambda)\psi_2\left[(1-\theta)\left(1-\tilde{p}_M^1\right) + \theta\int_{\tilde{p}_M^1}^1 \left(x - \tilde{p}_M^1\right)dx\right] \\ & + [1-(1-\lambda)\psi_2](1-\theta)\left(1-\tilde{p}_M^1+\tilde{p}_N^2\right)\left(1-\tilde{p}_M^1\right) \\ & + [1-(1-\lambda)\psi_2]\left[\theta\left(1-\tilde{p}_N^2\right) + (1-\theta)\int_{1-\tilde{p}_M^1+\tilde{p}_N^2}^1 \left(x-\tilde{p}_N^2\right)dx\right], \end{aligned} \tag{4.19}$$

and

$$\begin{aligned} \tilde{W}(\psi_2, \theta, \lambda) = \;& (1-\lambda)\psi_2\left[(1-\theta) + \theta\int_{\tilde{p}_M^1}^1 x dx\right] + [1-(1-\lambda)\psi_2](1-\theta)\left(1-\tilde{p}_M^1+\tilde{p}_N^2\right) \\ & + [1-(1-\lambda)\psi_2]\left[\theta + (1-\theta)\int_{1-\tilde{p}_M^1+\tilde{p}_N^2}^1 x dx\right]. \end{aligned} \tag{4.20}$$

The sales Gini coefficient in equilibrium is

$$\tilde{G}(\psi_2, \theta, \lambda) = \frac{1}{2} - \frac{D_N^2\left(\tilde{p}_N^2, \tilde{p}_M^1\right)}{D_M^1\left(\tilde{p}_M^1, \tilde{p}_N^2\right) + D_N^2\left(\tilde{p}_N^2, \tilde{p}_M^1\right)}. \tag{4.21}$$

Proposition 4.6. *When $\theta < \theta_0$ and $\psi_2 < \frac{1}{3(1-\lambda)}$, introducing mixed organic ranking can improve the search engine's immediate revenue and long-term growth, social welfare, and sales diversity concurrently in that (i) $\frac{\partial}{\partial\lambda}\Delta\tilde{\pi}_N(\psi_2, \theta, \lambda)|_{\lambda=0} > 0$; (ii) $\frac{\partial}{\partial\lambda}\tilde{CS}(\psi_2, \theta, \lambda)|_{\lambda=0} > 0$ (iii) $\frac{\partial}{\partial\lambda}\tilde{W}(\psi_2, \theta, \lambda)|_{\lambda=0} > 0$; and (iv) $\frac{\partial}{\partial\lambda}\tilde{G}(\psi_2, \theta, \lambda)|_{\lambda=0} < 0$.* Here, $\theta_0 \in \left(0, \frac{1}{2}\right)$ *solves the equation* $\frac{\partial}{\partial\lambda}\Delta\tilde{\pi}_N(\psi_2, \theta_0, \lambda)|_{\psi_2=\frac{1}{3(1-\lambda)}, \lambda=0} = 0$.

The proposition indicates that mixed organic ranking improves both profitability and efficiency in the cases when pure ranking is unable to engender satisfactory revenue, efficiency, and diversity. Recall the results from Corollary 4.1 and Propositions 4.2 through 4.4: When θ is small and ψ_2 is not too large, pure organic ranking cannot induce structural improvement in consumer surplus, social welfare, or sales diversity, while search engine revenue is extremely low. In this sense, Proposition 4.6 shows that introducing randomization serves as a good remedy to relieve the major drawback of the pure organic ranking mechanism.

Under pure organic ranking, as is discussed, when θ is small and ψ_2 is not too large, the niche firm's *promotive* incentive for winning a prominent sponsored position completely vanishes due to its huge disadvantage in market preference and the mainstream firm's unshakable prominence dominance in organic listing, which hurts the revenue contributed to the search engine. In contrast, mixed organic ranking gives the niche firm chances to occupy the prominent position in the organic list, which makes it profitable to win the prominent sponsored slot because the niche firm can exploit informational monopoly in these cases. In other words, introducing mixed ranking adds *preventive* incentive to the niche firm's sponsored bidding motivation. Essentially, as bidders become more comparable, they are induced to bid more aggressively, and thus a properly set randomization factor can improve the auctioneer's revenue. This rationale is along the same spirit of promoting disadvantageous players or handicapping advantageous players for competition purposes in existing studies (Liu et al., 2010).

On the other hand, the equilibrium consumer surplus and social welfare are improved as well. The reason is that the mixed ranking reduces the aforementioned two sources of efficiency loss. Occasional perturbation in organic rank directly improves firm N's exposure and thus effectively reduces the *informational* loss of consumer surplus and social welfare. Meanwhile, a smaller prominence difference induces more intense price competition so that the *competitive* efficiency loss is also well controlled. Given the increase in the surplus of the potential consumers and considering that slight perturbation of commercial websites' links for certain markets would have little impact on general search engine users (who are mainly interested in non-commercial websites), overall, introducing mixed organic ranking could benefit the search engine in the long run.

An interesting aspect of Proposition 4.6 is that welfare and revenue can be improved simultaneously so that the search engine's short-term and long-term interests are aligned, unlike most existing studies in which increasing revenue is often at the cost of welfare. The key to this result is the auto-balance between organic ranking and sponsored bidding. Although mixed ranking reduces the mainstream firm's organic exposure, it does not significantly decrease the mainstream firm's overall exposure because the mainstream firm regains essential prominence by winning the prominent sponsored position. Therefore, mixed ranking promotes the weak player's prominence at little cost to the stronger one's exposure, which leads to less total informational efficiency loss and thus higher consumer surplus and social welfare. In this sense, a mixed organic ranking mechanism allocates the total resource of consumer attention in a more effective way.

Previous discussion shows that it is hardly possible for small firms facing highly asymmetric market preference to prevail in terms of equilibrium sales amount. In fact, the organic list in the co-listing case (with pure ranking) performs as an *implicit* adjustment to promote the weak players, which has been shown to serve the moderately-weak better than the very-tail. To mitigate this flaw and better serve the tails, the mixed organic ranking *explicitly* adjusts the competence difference by directly promoting the weak in the organic list and effectively increasing its exposure. As a result, equilibrium sales diversity improves (the Gini coefficient decreases), and even the weak firms with very small θ manage to achieve higher market share than under pure ranking.

4.6 Conclusion

Studying the intriguing role of organic listing in search advertising, we take a different perspective by focusing on the effects of organic listing as a competing information source on advertisers' advertising strategies as well as the equilibrium outcomes. This study thus complements the existing literature in deepening the understanding of this issue. This study also provides implications for search engine designers and marketing managers.

First, we provide economic justification for the common practice of the co-listing structure (with both organic list and sponsored links) in the search industry. On the surface, organic lists provide potential advertisers with prominence for free, which reduces their bidding incentive for sponsored exposure and thus hurts search engine revenue. Nevertheless, as leading firms typically gain more free prominence and their bidding incentive would decline more than less competitive firms, organic listings could leverage the competence difference and help the less competitive advertisers win a better position in the sponsored lists, which increases information completeness and improves equilibrium consumer surplus, social welfare and sales diversity. In this sense, organic listing may sacrifice profitability from a myopic view, but it could accelerate the growth of the user base and result in long-term benefit.

Furthermore, we underscore the importance of keeping organic listing "organic": Organic ranking should be strictly protected from advertisers' manipulation. Throughout this study, we take a cautious approach that organic listing stays under full control of the search engine. Nevertheless, two alternatives sometimes seem appealing to different parties and can occasionally be observed in the industry: Search engines may be tempted to "sell" slots in organic listing, or a third party may be interested in offering the service of promoting firms' organic ranking. As we show, the beneficial effects of the co-listing structure root in that it improves the equilibrium information structure through adjusting advertisers' sponsored bidding incentives rather than impairing the objectivity of the organic listing. Selling organic positions violates this spirit. It may increase the search engine's short-term revenue, but would eventually hurt the long-term growth. This could be the reason why the leading search engine in China, Baidu, has received increasing criticism for its selling of top organic slots to paid advertisers (Webster, 2008). The second alternative, known as *search engine optimization* (SEO), may cause similar problems; meanwhile, advertisers' expenditure flows to SEO companies, which would be even worse from search engines' perspective. It explains the practice that all search engines keep their organic ranking mechanism private and constantly develop more sophisticated mechanisms by introducing new factors into the ranking rule so as to adapt to possible SEO manipulation.

Meanwhile, we also suggest that while keeping organic listing unaltered from advertisers, the organic ranking mechanism could be further improved under certain cases. As we pinpoint, the drawbacks of popularity-based organic ranking arise in the presence of highly asymmetric market preference. We propose that slightly mixing the organic ranks of targeted commercial websites probabilistically may help improve the equilibrium outcomes for those markets, increasing direct revenue, consumer surplus, welfare, and diversity at the same time. This possible direction calls for novel algorithm designs to incorporate strategic perturbation into the organic ranking methods. Possible implementations might include randomizing between regular and reverse ordering of targeted advertisers with certain probability, or introducing additional factors beyond website relevance into the ranking score.

In addition to the implications for search engine designers, our study also provides rationale for marketing managers in dealing with online search advertising. Advertisers whose websites have already been placed in prominent positions in organic lists may wonder whether it is still worth investing in sponsored bidding. We suggest that sponsored bidding is rewarding when they have salient advantage in market preference, and dominating exposure in both lists maximizes their profitability. When market preference is relatively diversified, advertisers whose organic position is not satisfactory should bid aggressively to win the top sponsored slots because the marginal benefit of increasing sponsored exposure is fairly high in this case. In contrast, for very niche firms or individual sellers without sufficient competence in market preference, although it may seem appealing to expose themselves via this new advertising medium, excessive spending in sponsored bidding may have a low return on investment, because even a very prominent sponsored position with high click-through rate may bring high advertising bills but not commensurable profits.

This work triggers interesting directions for future research. For example, as we suggest that organic ranking could be further improved, especially for highly asymmetric markets, it leads to a challenging yet important research direction: the optimal design of organic ranking mechanism, taking into consideration advertisers' reaction in both sponsored bidding competition and product market competition. Notice that the optimal organic ranking mechanism should depend on the competitive situation among advertisers, which brings challenges for design science to devise novel algorithms to implement such situation-dependent dynamic ranking of organic links.

Appendices

Appendix A

Proofs for Chapter 2

Proof of Proposition 2.1

Denote $F_i(p)$ as $F_i^-(p)$ for $p \in [\bar{p}_{i+1}, \bar{p}_i)$ and as $F_i^+(p)$ for $p \in [\bar{p}_i, \bar{p}_{i-1}]$.

(i) First, $F_i(\cdot)$, $i = 1, ..., n$, is a well-defined cumulative distribution function. Notice that all supports are well defined because $0 < k_i < 1$, $i = 1, ..., n-1$, and thus $\{\bar{p}_i\}_{i=1}^n$ is positive and monotonically decreasing.

Each $F_i^-(\cdot)$ or $F_i^+(\cdot)$ is strictly increasing within its support, and $F_i^-(\bar{p}_i) = F_i^+(\bar{p}_i)$ because

$$1 - F_i^-(\bar{p}_i) = \frac{\bar{p}_{i+1}}{\bar{p}_i} = k_i = \frac{\alpha_{i-1}}{\alpha_i(1-\alpha_{i-1})}(\frac{1}{k_{i-1}} - 1) = \frac{\alpha_{i-1}}{\alpha_i(1-\alpha_{i-1})}\frac{(\bar{p}_{i-1} - \bar{p}_i)}{\bar{p}_i} = 1 - F_i^+(\bar{p}_i).$$

Therefore, each $F_i(\cdot)$ is strictly increasing in its entire support. Since $F_i(\bar{p}_{i+1}) = 0$ and $F_i(\bar{p}_{i-1}) = 1$, $F_i(\cdot)$ is well defined.

(ii) Next, we show that each position i yields a constant expected profit π_i within the entire support $[\bar{p}_{i+1}, \bar{p}_{i-1}]$.

If the firm in position i ($i = 2, ..., n-1$) prices at $p \in [\bar{p}_i, \bar{p}_{i-1}]$, it only competes with the firm in position $i-1$ for the demand $\alpha_i\beta_i$. (Recall that $\beta_i = \prod_{j=1}^{i-1}(1-\alpha_j)$.) Thus, the expected profit is $\pi_i^+(p) = [1 - F_{i-1}^-(p)] \cdot p \cdot \alpha_i\beta_i \equiv \alpha_i\beta_i\bar{p}_i$. If it prices in the lower interval $[\bar{p}_{i+1}, \bar{p}_i]$, it will win over the firm above and capture the demand $\alpha_i\beta_i$, and meanwhile compete with the firm below for the demand $\alpha_{i+1}\beta_{i+1}$, which leads to the expected profit:

$$\begin{aligned} \pi_i^-(p) &= p \cdot \alpha_i\beta_i + [1 - F_{i+1}^+(p)] \cdot p \cdot \alpha_{i+1}\beta_{i+1} \\ &\equiv \alpha_i\beta_i\bar{p}_i. \end{aligned}$$

Therefore, $\pi_i^+(p) = \pi_i^-(p)$; that is, the expected profit remains constant over the entire support.

The firm at position n competes only with the firm above it. A calculation similar to that for the above $\pi_i^+(p)$ shows a constant expected profit $\beta_n \bar{p}_n$. Similarly, we can verify the constant expected profit for the firm at position 1.[1]

(iii) Now we verify that no unilateral profitable deviation exists by pricing outside the given support.

Suppose the firm in position i (≥ 3) prices at $p > \bar{p}_{i-1}$. If $p < \bar{p}_{i-2}$, it faces a competition with the two firms above, and the expected profit is

$$\pi_i(p) = [1 - F_{i-1}^+(p)][1 - F_{i-2}^-(p)] \cdot p \cdot \alpha_i \beta_i = \alpha_i \beta_i \frac{\alpha_{i-2}(\bar{p}_{i-2} - p)}{\alpha_{i-1}(1 - \alpha_{i-2})} \frac{\bar{p}_{i-1}}{p},$$

which is strictly decreasing in p; that is, $\pi_i(p) < \pi_i(\bar{p}_{i-1}), \forall p \in (\bar{p}_{i-1}, \bar{p}_{i-2})$. If $p \geq \bar{p}_{i-2}$, then the firm would have zero demand (and thus zero profit) because the firm right above it charges a lower price for sure. The firms at positions 1 and 2 will clearly not charge a price higher than $\bar{p}_1$, since $\bar{p}_1 = w$, consumers' willingness-to-pay. Thus, there is no unilateral profitable deviation by pricing higher beyond the given support.

Suppose the firm in position i ($\leq n-2$) prices at $p < \bar{p}_{i+1}$. If $p \in [\bar{p}_{i'+1}, \bar{p}_{i'}]$ ($i' \geq i+1$), it could capture all consumers who stop sampling at any position from i to $i' - 1$. We denote the amount of those consumers as $D_{ii'}$. In addition, it competes with the firms in positions i' and $i' + 1$ for consumers who stop sampling at positions i' and $i' + 1$. Thus, the expected profit can be written as

$$\begin{aligned} \pi_i(p) &= D_{ii'}p + [1 - F_{i'}^-(p)]p\alpha_{i'}\beta_{i'} + [1 - F_{i'}^-(p)][1 - F_{i'+1}^+(p)]p\alpha_{i'+1}\beta_{i'+1} \\ &= D_{ii'}p + [1 - F_{i'}^-(p)] \cdot \alpha_{i'}\beta_{i'}\bar{p}_{i'} \\ &= D_{ii'}p + \alpha_{i'}\beta_{i'}\bar{p}_{i'}\tfrac{\bar{p}_{i'+1}}{p}, \end{aligned}$$

where the second and third steps are derived by substituting in $F_{i'+1}^+(p)$ and $F_{i'}^-(p)$ from Equation (2.1), respectively. Note that $D_{ii'} \geq \alpha_{i'-1}\beta_{i'-1} \geq \alpha_{i'}\beta_{i'}$, where the last inequality is due to the assumption $\alpha_i \geq \alpha_{i+1}(1-\alpha_i)$. Therefore, $\pi_i(\bar{p}_{i'}) - \pi_i(\bar{p}_{i'+1}) = (\bar{p}_{i'} - \bar{p}_{i'+1})(D_{ii'} - \alpha_{i'}\beta_{i'}) \geq 0$. Also noticing that $\pi_i(p)$ is convex, we can conclude that $\pi_i(p) < \pi_i(\bar{p}_{i'})$,

[1]The only exception is that there will be a downward jump in expected profit at the upper bound $\bar{p}_1$ for the firm in position 2, which is incurred by the mass point in $F_1(\cdot)$ at $\bar{p}_1$. However, a jump down at one single point with zero probability measure does not affect the actual profit.

$\forall p \in (\bar{p}_{i'+1}, \bar{p}_{i'})$. Iteratively, we get $\pi_i(p) \leq \pi_i(\bar{p}_{i+1})$, $\forall p < \bar{p}_{i+1}$. For firms at position $n-1$ and n, clearly, it is unprofitable to price below $\bar{p}_n$, since otherwise the profit margin would decrease without an increase in demand. Hence, there is no unilateral profitable deviation by pricing below the given support.

Combining (ii) and (iii), we conclude that given other firms' strategies, each firm is indifferent in pricing over the given support and has no profitable deviation. Therefore, the price strategy described in the proposition is a mixed-strategy equilibrium.

Proof of Corollary 2.2

For $i = 2, ..., n-1$, denote $E_i^-(p_i)$ and $E_i^+(p_i)$ as the price expectations from position i, *conditional on* $p_i \in [\bar{p}_{i+1}, \bar{p}_i]$ and *conditional on* $p_i \in [\bar{p}_i, \bar{p}_{i-1}]$, respectively. Also denote $f_i^-(\cdot)$ and $f_i^+(\cdot)$ as the corresponding conditional probability density functions. According to Equation (2.1), both $f_i^-(p)$ and $f_{i+1}^+(p)$ take the form of $\frac{K}{p^2}$ on the same support $[\bar{p}_{i+1}, \bar{p}_i]$ and thus must be identical. (It can also be calculated that K is $\frac{\bar{p}_i \bar{p}_{i+1}}{\bar{p}_i - \bar{p}_{i+1}}$ in the former and is $\frac{\alpha_i}{\alpha_{i+1}(1-\alpha_i)} \frac{\bar{p}_i \bar{p}_{i+1}}{\bar{p}_{i+2}}$ in the latter. They are the same due to Equation (2.2) and Equation (2.3)). Therefore, $E_i^-(p_i) = E_{i+1}^+(p_{i+1})$. Notice that the expected price from the ith position $E(p_i)$ is the weighted average of $E_i^-(p_i)$ and $E_i^+(p_i)$, and that from $i+1$th position $E(p_{i+1})$ is the weighted average of $E_{i+1}^-(p_{i+1})$ and $E_{i+1}^+(p_{i+1})$. Since $E_i^-(p_i) = E_{i+1}^+(p_{i+1})$ and $E_i^+(p_i) > E_{i+1}^-(p_{i+1})$, $E(p_i) > E(p_{i+1})$. Similar arguments apply to position 1 and position n.

Proof of Proposition 2.2

First, we show that given consumers' search strategies (i.e., non-shoppers' optimal stopping prices r_1^* and r_2^*), and other firms' pricing strategies as Equation (2.8), no firm has profitable deviation in pricing. When pricing any $p \in [\bar{p}_2, \bar{p}_1]$, a firm achieves a constant expected profit

$$\pi(p) = \frac{1}{3}\left[\alpha + \alpha(1-\alpha)\right] p + (1-\alpha)^2 p \left[1 - F(p)\right]^2 \equiv \frac{1}{3}\left[\alpha + \alpha(1-\alpha)\right] \bar{p}_1$$

by substituting in $F(p)$. If a firm deviates to charge a price equal to r_1^*, it forgoes the type-2 consumers while still retaining the type-1 consumers. Its expected profit would be

$$\pi^{'}\left(r_1^*\right)=\frac{1}{3}\alpha r_1^*.$$

Notice that $r_2^* > \frac{1}{2-\alpha}r_1^*$, which implies $\pi^{'}\left(r_1^*\right) < \frac{1}{3}\left[\alpha+\alpha\left(1-\alpha\right)\right]\bar{p}_1$. Since any price $p \in \left(r_2^*, r_1^*\right)$ yields an even lower expected profit and any price $p > r_1^*$ results in zero profit, there is no profitable deviation from pricing above $\bar{p}_1$. Also, there is no profitable deviation from underpricing, because any $p < \bar{p}_2$ yields a lower profit level than $\frac{1}{3}\left[\alpha+\alpha\left(1-\alpha\right)\right]\bar{p}_1$.

Second, we show that given firms' pricing strategies in Equation (2.8), r_1^* and r_2^* are rational. By Equation (2.7), consumers' rationality requires

$$\begin{cases}\int_{\bar{p}_2}^{r_2^*}\left(r_2^*-p\right)dF\left(p\right)-k^{'}=0\\ \int_{\bar{p}_2}^{r_1^*}\left(r_1^*-p\right)dF\left(p\right)-k=0,\end{cases}$$

which is ensured by $\int_{\bar{p}_2}^{\bar{p}_1}F\left(p\right)dp=k^{'}$ and $r_1^*=r_2^*+k-k^{'}$. Notice that since $r_2^*<r_1^*<1$, it is rational for both types of non-shoppers to enter the market.

Altogether, the strategy profile specified in Proposition 2.2 is an equilibrium.

Proof of Proposition 2.3

First, $F\left(p\right)$ in Equation (2.9) is a well-defined cumulative distribution function. We can verify that $F\left(\bar{p}_4\right)=0$, $F\left(\bar{p}_1\right)=1$, and F is increasing in the two segments of the support. Also, $F\left(\bar{p}_3\right)=F\left(\bar{p}_2\right)=1-\phi$. To see this, first notice that ϕ is the solution to the quadratic equation

$$\bar{p}_3\left[\alpha+\alpha\left(1-\alpha\right)\left(1+\phi+\phi^2\right)+3\left(1-\alpha\right)^2\phi^2\right]=\alpha\bar{p}_1. \tag{A.1}$$

Let the left-hand side of Equation (A.1) be $f\left(\phi\right)$. The condition $\frac{\alpha}{3-2\alpha}r_1^*<r_2^*<\frac{1}{2-\alpha}r_1^*$ ensures that $f\left(0\right)<\alpha\bar{p}_1$ and $f\left(1\right)>\alpha\bar{p}_1$. Also, the quadratic coefficient of $f\left(\phi\right)$ is positive. Hence, $\phi\in\left(0,1\right)$ is well defined. Therefore, substituting Equation (A.1) and the definition of $\bar{p}_2$ into the first and second equations of (2.9), respectively, we have $F\left(\bar{p}_3\right)=F\left(\bar{p}_2\right)=1-\phi$. The price support is also well defined. $\bar{p}_2>\bar{p}_3$ because $\bar{p}_3=\frac{\alpha\bar{p}_1}{\alpha+\alpha(1-\alpha)(1+x+x^2)+3(1-\alpha)^2x^2}$ by Equation (A.1) and its denominator is greater than that of $\bar{p}_2$

$\left(\text{i.e., } \alpha + \alpha(1-\alpha)\left[1+x+x^2\right] + 3(1-\alpha)^2 x^2 - \alpha - 3(1-\alpha)x^2 = \alpha(1-\alpha)\left[\left(1-x^2\right) + x(1-x)\right] > 0\right)$. Similarly, comparing the denominators of $\bar{p}_3$ and $\bar{p}_4$, we have $\bar{p}_3 > \bar{p}_4$.

Next, we show that given consumers' optimal stopping prices (i.e., r_1^* and r_2^*), and other firms' pricing strategies, no firm has profitable deviation in pricing. When pricing $p \in [\bar{p}_4, \bar{p}_3]$, a firm's expected profit function can be written as

$$\pi(p) = \frac{1}{3}\alpha p + \frac{1}{3}\alpha(1-\alpha)\left[1 + (1-F(r_2^*)) + (1-F(r_2^*))^2\right] p + (1-\alpha)^2\left[1-F(p)\right]^2 p,$$

where the first term on RHS is the expected revenue from the type-1 consumers, the second term is that from the type-2 consumers, and the third term is that from shoppers. In particular, the expected revenue from the type-2 consumers consists of three parts: those who inspect this firm first and stop there $\left(\frac{1}{3}\alpha(1-\alpha)\right)$, those who inspect another firm first but find its price exceeding r_2^* and continue to inspect this firm $\left(2 \times \frac{1}{3}\alpha(1-\alpha)(1-F(r_2^*)) \times \frac{1}{2}\right)$, and those who inspect the other two firms first but find both their prices exceeding r_2^* and continue to inspect this firm $\left(2 \times \frac{1}{3}\alpha(1-\alpha)(1-F(r_2^*))\frac{1}{2}(1-F(r_2^*))\right)$. Substituting $F(p)$ from the first equation of Equation (2.9) (with $F(r_2^*) = 1-\phi$) into the above profit function, we have $\pi(p) \equiv \frac{1}{3}\alpha\bar{p}_1$, $\forall p \in [\bar{p}_4, \bar{p}_3]$. Similarly, when pricing $p \in [\bar{p}_2, \bar{p}_1]$, a firm's expected profit function can be written as

$$\pi(p) = \frac{1}{3}\alpha p + (1-\alpha)\left[1-F(p)\right]^2 p,$$

where the first term on RHS is the expected revenue from the type-1 consumers and the second term is that from the type-2 consumers and the shoppers. Substituting in $F(p)$ from the second equation of Equation (2.9), we have $\pi(p) \equiv \frac{1}{3}\alpha\bar{p}_1$, $\forall p \in [\bar{p}_2, \bar{p}_1]$. Therefore, pricing within the support leads to a constant expected profit. In addition, any price $p \in (\bar{p}_3, \bar{p}_2)$ yields a lower profit than $\pi(\bar{p}_2)$. Any price $p > \bar{p}_1$ yields zero profit. Any price $p < \bar{p}_4$ yields a lower profit than $\pi(\bar{p}_4)$. Therefore, there is no profitable unilateral deviation for any firm.

Given firms' pricing strategies, r_1^* and r_2^* are rational because the rationality requirements

$$\begin{cases} \int_{\bar{p}_4}^{\bar{p}_3} (r_2^* - p)\, dF(p) = k' \\ \int_{\bar{p}_4}^{\bar{p}_3} (r_1^* - p)\, dF(p) + \int_{\bar{p}_2}^{\bar{p}_1} (r_1^* - p)\, dF(p) = k \end{cases}$$

are ensured by Equation (2.10). Notice that since $r_2^* < r_1^* < 1$, it is rational for both types to enter the market.

Altogether, the strategy profile specified in Proposition 2.3 is an equilibrium.

Proof of Proposition 2.4

To prove by contradiction, suppose that $k_1 \geq k_3$ and $k_1^{'} \geq k_3^{'}$ and that the price pattern in Equation (2.1) is an equilibrium. Then for both the type-1 and the type-2 consumers, inspecting the first position is dominated by inspecting the third one, because the third position offers a lower price for sure, while it does not incur a higher search cost. Thus, the rational search strategy should not involve inspecting the first position. In this case, the first firm has zero profit by charging $p \in [\bar{p}_2, \bar{p}_1]$ and will deviate to charge a lower price $p^{'} \in [\bar{p}_3, \bar{p}_2)$ which gives positive expected profit. Also, given firm 3's price support, no matter what search order consumers choose between firm 2 and firm 3, it is suboptimal for firm 2 to price $p \in (\bar{p}_2, \bar{p}_1)$. In any case, the given pricing pattern cannot hold in equilibrium if $k_1 \geq k_3$ and $k_1^{'} \geq k_3^{'}$.

Proof of Proposition 2.5

First, we show that given consumers' search strategies and other firms' pricing strategies, no firm has profitable unilateral deviation. For firm 1, charging $\forall p \in [\bar{p}_2, \bar{p}_1]$ gives a constant expected profit

$$\pi_1(p) \quad = \quad \alpha p + \alpha(1-\alpha) p [1 - F_2(p)] \equiv \alpha \bar{p}_1$$

by substituting in firm 2's pricing strategies. Similarly, we can show that firm 2 has a constant expected profit by charging any price within $[\bar{p}_3, \bar{p}_1)$ because

$$\begin{array}{ll} \pi_2(p) = \alpha(1-\alpha) p [1 - F_1(p)] \equiv \alpha(1-\alpha) \bar{p}_2 & p \in (\bar{p}_2, \bar{p}_1) \\ \pi_2(p) = \alpha(1-\alpha) p + (1-\alpha)^2 p [1 - F_3(p)] \equiv \alpha(1-\alpha) \bar{p}_2 & p \in [\bar{p}_3, \bar{p}_2]. \end{array}$$

Notice that $\pi_2(\bar{p}_1) < \alpha(1-\alpha)\bar{p}_2$ because of the mass point in F_1 at the upper bound. Nevertheless, since F_2 places no mass on $\bar{p}_1$, firm 2's expected profit is unaffected. Firm 3 also has constant profit by charging any price within its support:

$$\pi_3(p) = (1-\alpha)^2 p [1 - F_2(p)] \equiv (1-\alpha)^2 \bar{p}_3 \quad p \in [\bar{p}_3, \bar{p}_2].$$

If firm 3 deviates to charge $p \in (\bar{p}_2, \bar{p}_1)$, its profit would be

$$\pi_3^{'}(p) = (1-\alpha)^2 p \left[1 - F_2(p)\right]\left[1 - F_1(p)\right] = (1-\alpha)\,\bar{p}_2 \frac{(\bar{p}_1 - p)}{p},$$

which is decreasing in p. Therefore, pricing above $\bar{p}_2$ is not profitable for firm 3. Also, firm 1 cannot achieve a higher profit by deviating to charge $p \in [\bar{p}_3, \bar{p}_2)$. To see this, first notice that after inspecting the first position, type-2 consumers' search strategy, $d(p, \{2, 3\})$, is to inspect the second position if the price learned from firm 1 exceeds r_2, and to stop and buy from firm 1 otherwise. Here, r_2 $(\bar{p}_3 < r_2 < \bar{p}_2)$ is defined by $\int_{\bar{p}_3}^{r_2} (r_2 - p)\, dF_2(p) = k_2^{'}$. When $p \in (r_2, \bar{p}_2)$, type-2 consumers continue to inspect the second position, and firm 1's profit would be

$$\begin{aligned}\pi_1^{'}(p) &= \alpha p + \alpha(1-\alpha)\,p\left[1 - F_2(p)\right] + (1-\alpha)^2 p\left[1 - F_2(p)\right]\left[1 - F_3(p)\right] \\ &= \alpha p + \alpha(1-\alpha)\,\bar{p}_3 + (1-\alpha)^2\,\bar{p}_3 \frac{\alpha(\bar{p}_2 - p)}{(1-\alpha)p},\end{aligned}$$

which is convex in p. Since $\pi_1^{'}(\bar{p}_3) = \bar{p}_3 < \alpha\bar{p}_1 = \pi_1^{'}(\bar{p}_2)$, we can conclude that $\pi_1^{'}(p) < \pi_1(\bar{p}_2)$ for $\forall p \in (r_2, \bar{p}_2)$. When $p \in [\bar{p}_3, r_2]$, type-2 consumers stop searching and buy from firm 1, and firm 1's profit function thus becomes

$$\pi_1^{''}(p) = [\alpha + \alpha(1-\alpha)]p + (1-\alpha)^2 p\left[1 - F_2(p)\right]\left[1 - F_3(p)\right],$$

which again is convex in p. We can verify that $\pi_1^{''}(\bar{p}_3) < \pi_1(\bar{p}_2) = \alpha\bar{p}_1$. In addition, the condition on $k_2^{'}$ ensures that $\pi_1^{''}(r_2) < \pi_1(\bar{p}_2)$. This is because $(r_2 - \bar{p}_3) - \bar{p}_3 \ln\frac{r_2}{\bar{p}_3} = k_2^{'}$ by substituting F_2 into the definition of r_2, which implies that r_2 increases with $k_2^{'}$. When $k_2^{'} < (\gamma^* - \bar{p}_3) - \bar{p}_3 \ln\frac{\gamma^*}{\bar{p}_3}$, $r_2 < \gamma^*$. Notice that γ^* solves the equation $\pi_1^{''}(\gamma^*) = \alpha\bar{p}_1$. Hence, $\pi_1^{''}(r_2) < \pi_1(\bar{p}_2)$, and thus $\pi_1^{''}(p) < \pi_1(\bar{p}_2)$ for $\forall p \in [\bar{p}_3, r_2]$. Therefore, firm 1 has no profitable deviation by underpricing. In addition, for all firms, charging any price greater than $\bar{p}_1$ leads to zero profit, and charging any price below $\bar{p}_3$ leads to lower profit. Altogether, all firms achieve constant profit within their given price supports and no firm has profitable deviation outside their price supports. Therefore, given consumers' search strategies, firms' pricing strategies form an equilibrium.

Next, we show that given firms' pricing strategies, the search strategies of non-shoppers, listed in Table 2.1 are rational. Consider type-1 consumers' strategies. When making decision $d(z,\{1\})$, if $z \leq r_1$, by Equation (2.6), the expected gain equals

$$EG(1;z,\{1\}) = \int_{\bar{p}_2}^{z} (z-p)\, dF_1(p) - k_1 \leq 0$$

because r_1 is defined as $\int_{\bar{p}_2}^{r_1} (r_1 - p)\, dF_1(p) = k_1$ and $\int_{\bar{p}_2}^{z} (z-p)\, dF_1(p)$ is increasing in z. $d(z,\{2\})$ is to stop when $z \leq \bar{p}_1$ because, by the definition of $\bar{p}_1$,

$$\begin{aligned} EG(2;\bar{p}_1,\{2\}) &= \int_{\bar{p}_3}^{\bar{p}_1} (\bar{p}_1 - p)\, dF_2(p) - k_2 \\ &= \left[1 + \frac{\alpha \ln \alpha}{1+\alpha(1-\alpha)} - \frac{\ln(1+\alpha(1-\alpha))}{1-\alpha}\right] \bar{p}_1 - k_2 \leq 0. \end{aligned}$$

Similarly, for $d(z,\{3\})$, by the condition $k_3 \geq k_3' > \left[1 + \frac{\alpha \ln \alpha}{(1-\alpha)[1+\alpha(1-\alpha)]}\right] \bar{p}_1$, we can check that

$$EG(3;\bar{p}_1,\{3\}) = \int_{\bar{p}_3}^{\bar{p}_2} (\bar{p}_1 - p)\, dF_3(p) - k_3 = \left[1 + \frac{\alpha \ln \alpha}{(1-\alpha)[1+\alpha(1-\alpha)]}\right] \bar{p}_1 - k_3 < 0.$$

For $d(z,\{2,3\})$, notice that any $z \leq 1$, by Equation (2.6),

$$EG(2;z,\{2,3\}) = \int_{\bar{p}_3}^{z} (z-p)\, dF_2(p) - k_2 = EG(2;z,\{2\}).$$

The reason is that the third term on RHS of Equation (2.6) equals zero because the price p learned from the second firm is less than $\bar{p}_1$; thus, the lowest price $z' = \min\{z,p\} \leq \bar{p}_1$, and the decision $d(z',\{3\})$ is to stop. By similar arguments, $EG(3;z,\{2,3\}) = EG(3;z,\{3\})$. Therefore, comparing $EG(2;z,\{2,3\})$ and $EG(3;z,\{2,3\})$ is equivalent to comparing $EG(2;z,\{2\})$ and $EG(3;z,\{3\})$. Notice that the condition $k_3 - k_2 > \bar{p}_3 \frac{\alpha}{1-\alpha} \ln \alpha + \frac{\bar{p}_1}{1-\alpha} \ln(1+\alpha(1-\alpha))$ ensures that $EG(2;z,\{2\}) > EG(3;z,\{3\})$ for $\forall z \in [\bar{p}_1, 1]$. Therefore, if $z > \bar{p}_1$, it is optimal to inspect the second position; if $z \leq \bar{p}_1$, it is optimal to stop because both $EG(2;z,\{2\})$ and $EG(3;z,\{3\})$ are non-positive. For the decision $d(z,\{1,3\})$, we can similarly show that $EG(1;z,\{1,3\}) = EG(1;z,\{1\})$ and $EG(3;z,\{1,3\}) = EG(3;z,\{3\})$. Notice that the condition $k_1 < \frac{\alpha(1-\alpha) - \ln(1+\alpha(1-\alpha))}{1+\alpha(1-\alpha)} \bar{p}_1$ ensures that $EG(1;\bar{p}_1,\{1\}) > 0$ and $\bar{p}_2 < r_1 < \bar{p}_1$ by the definition of r_1. Recall that

$EG(3;z,\{3\}) < 0$ if $z \leq \bar{p}_1$. We can also conclude that for $\forall z \geq \bar{p}_1$, $EG(1;z,\{1\}) > EG(3;z,\{3\})$. Therefore, the decision $d(z,\{1,3\})$ listed in Table 2.1 is rational. For $d(z,\{1,2\})$, when $z \leq \bar{p}_2$, $EG(1;z,\{1,2\}) = EG(1;z,\{1\}) < 0$ and $EG(2;z,\{1,2\}) = EG(2;z,\{2\}) < 0$, and thus to stop is optimal. Finally, we need to check the first-step decision $d(1,\{1,2,3\})$. We need to compare $EG(1;1,\{1,2,3\})$, $EG(2;1,\{1,2,3\})$, and $EG(3;1,\{1,2,3\})$. Based on the subsequent strategies, we can conclude that $EG(1;1,\{1,2,3\}) = EG(1;1,\{1\}) > 0$, which indicates that to enter the market is always rational. Also, $EG(3;1,\{1,2,3\}) = EG(3;1,\{3\})$, and we have $EG(1;1,\{1,2,3\}) > EG(3;1,\{1,2,3\})$ because $EG(1;1,\{1\}) > EG(3;1,\{3\})$, as is already shown. We can write the expected gain of first inspecting the second position as

$$\begin{aligned}
EG(2;1,\{1,2,3\}) &= \int_{\bar{p}_3}^{\bar{p}_1} (1-p)\, dF_2(p) - k_2 + \int_{r_1}^{\bar{p}_1} \left[\int_{\bar{p}_2}^{p} (p-x)\, dF_1(x) - k_1 \right] dF_2(p) \\
&< (1-\bar{p}_1) + \left[\int_{\bar{p}_3}^{\bar{p}_1} (\bar{p}_1 - p)\, dF_2(p) - k_2 \right] + \int_{r_1}^{\bar{p}_1} \left[\int_{\bar{p}_2}^{\bar{p}_1} (\bar{p}_1 - x)\, dF_1(x) - k_1 \right] dF_2(p) \\
&< (1-\bar{p}_1) + 0 + \left[\int_{\bar{p}_2}^{\bar{p}_1} (\bar{p}_1 - x)\, dF_1(x) - k_1 \right] = EG(1;1,\{1,2,3\}).
\end{aligned}$$

Therefore, it is optimal to start searching from the first position rather than the second or the third. In a similar manner, we can check that all strategies of the type-2 consumers listed in Table 2.1 are rational.

Altogether, under the given parametric conditions, the strategy profile specified in Proposition 2.5 is a rational-expectations equilibrium.

Appendix B

Proofs for Chapter 3

Proof of Lemma 3.1

(i) If $\alpha w > c$ and such that $m = \alpha w$, $F_H(p) = \frac{p-\alpha w}{p-c}$ with a mass point at the upper bound w (with probability $\frac{\alpha w-c}{w-c}$), and $F_L(p) = \frac{p-\alpha w}{(1-\alpha)p}$. We can verify that firms have no profitable deviation. This is because, according to firms' payoff function (denote $\pi_i^j(p)$ as the expected profit of firm i in position j charging price p),

$$\begin{array}{rcl}\pi_H^1(p) & = & \alpha p + (1-\alpha)p[1-F_L(p)] \\ \pi_L^2(p) & = & (1-\alpha)(p-c)[1-F_H(p)]\end{array} \tag{B.1}$$

both firms achieve a constant expected profit level within the support (i.e., $\pi_H^1(p) \equiv \alpha w$, $\pi_L^2(p) \equiv (1-\alpha)(\alpha w - c)$, $\forall p \in [\alpha w, w)$).

If $\alpha w \leq c$ and such that $m = c$, H takes advantage of its low cost, charging $p_H = c$ for sure ($F_H(p) \equiv 1$, $\forall p \in [c, w]$), which gives L zero profit. Meanwhile, L plays mixed strategy $F_L(p) = \frac{p-c}{(1-\alpha)p}$ such that H has no profitable deviation, because $\pi_H^1(p) \equiv c$, $\forall p \in [c, w)$, according to Eq.(B.1).

Additionally, in both cases, neither firm has incentive to charge higher than w or lower than m, since the former leads to no purchase and zero profit and the latter is dominated by charging m.

(ii) Similarly, we can verify that both firms have constant profit level within the support:

$$\begin{array}{l}\pi_H^2(p) = (1-\alpha)p[1-F_L(p)] \equiv (1-\alpha)[\alpha(w-c)+c] \\ \pi_L^1(p) = \alpha(p-c) + (1-\alpha)(p-c)[1-F_H(p)] \equiv \alpha(w-c)\end{array} \tag{B.2}$$

Again, we can check that there is no profitable deviation by charging outside the given support.

Proof of Proposition 3.1

According to Table 3.1,

$$\Delta\pi_L = \pi_L^1 - \pi_L^2 = \alpha(w-c) - (1-\alpha)(m-c) \tag{B.3}$$

$$\Delta\pi_H = \pi_H^1 - \pi_H^2 = m - (1-\alpha)(c + \alpha(w-c)) \tag{B.4}$$

For firm L, if $\alpha w \le c$, $\Delta\pi_L = \alpha(w-c) > 0$; if $\alpha w > c$, $\Delta\pi_L = \alpha(\alpha w - c) + (1-\alpha)c > 0$. Therefore, $\Delta\pi_L > 0$.

For firm H, if $\alpha w \le c$, $\Delta\pi_H = \alpha(2-\alpha)c - \alpha(1-\alpha)w$, and thus $\Delta\pi_H < 0$ iff $c < \frac{1-\alpha}{2-\alpha}w$. If $\alpha w > c$, $\Delta\pi_H = \alpha^2 w - (1-\alpha)^2 c$, and thus $\Delta\pi_H < 0$ iff $c > \frac{\alpha^2}{(1-\alpha)^2}w$. Notice at $\alpha = \frac{3-\sqrt{5}}{2}$, $\frac{1-\alpha}{2-\alpha}w = \frac{\alpha^2}{(1-\alpha)^2}w = \alpha w$ (three lines intersect at one point). Therefore, $\Delta\pi_H < 0$ iff $\frac{\alpha^2}{(1-\alpha)^2}w < c < \frac{1-\alpha}{2-\alpha}w$.

Proof of Proposition 3.2

By Eq.(B.3) and Eq.(B.4), if $\alpha w \le c$, $\Delta\pi_H - \Delta\pi_L = \alpha[(3-\alpha)c - (2-\alpha)w]$, and thus $\Delta\pi_H > \Delta\pi_L$ iff $c > \frac{2-\alpha}{3-\alpha}w$. If $\alpha w > c$, $\Delta\pi_H - \Delta\pi_L = (-\alpha^2 + 4\alpha - 2)c$, and thus $\Delta\pi_H > \Delta\pi_L$ iff $2 - \sqrt{2} < \alpha$ (< 1). Notice the line $\frac{2-\alpha}{3-\alpha}w$ intersects with line αw at $\alpha = 2 - \sqrt{2}$. Therefore, $\Delta\pi_H > \Delta\pi_L$ if $\alpha > 2 - \sqrt{2}$ or $c > \frac{2-\alpha}{3-\alpha}w$. By Proposition 3.1, $\Delta\pi_L > 0$ and $\Delta\pi_H < 0$ only when $\frac{\alpha^2}{(1-\alpha)^2}w < c < \frac{1-\alpha}{2-\alpha}w$. So, when $\alpha > 2-\sqrt{2}$ or $c > \frac{2-\alpha}{3-\alpha}w$, $b_H = \Delta\pi_H > \Delta\pi_L = b_L > 0$. When $\alpha < 2 - \sqrt{2}$ and $c < \frac{2-\alpha}{3-\alpha}w$, $\Delta\pi_H < \Delta\pi_L$, and thus $b_H = \max\{0, \Delta\pi_H\} < \Delta\pi_L = b_L$.

Proof of Proposition 3.3

(i) In this case, L wins the first position. According to Eq.(3.2), $F_L(p) < F_H(p), \forall p \in (c+\alpha(w-c), w]$, which means that the price by firm L first order stochastically dominates the price by firm H. Thus, $E(p_L) > E(p_H)$.

(ii) In this case, H wins the first position. If $\alpha w \le c$, by Eq.(3.1), H plays pure strategy at c while L mixes over $[c, w]$. Clearly, $E(p_H) < E(p_L)$.

If $\alpha w > c$, according to Eq.(3.1),

$$E(p_H) - E(p_L) = \int_{\alpha w}^{w} F_L(p) - F_H(p)dp = w[(\alpha - \frac{c}{w})\ln\frac{1-c/w}{\alpha - c/w} + \alpha + \frac{\alpha}{1-\alpha}\ln\alpha]$$

Define the above as $f(\alpha, c, w)$, and notice that $\frac{\partial f}{\partial \alpha} = w[\ln\frac{1-c/w}{\alpha-c/w} + \frac{\ln\alpha}{(1-\alpha)^2} + \frac{1}{1-\alpha}]$ and $\frac{\partial^2 f}{\partial \alpha^2} = w[-\frac{1}{\alpha-c/w} + \frac{1}{\alpha(1-\alpha)^2} + \frac{2\ln\alpha}{(1-\alpha)^3} + \frac{1}{(1-\alpha)^2}]$. Apply Taylor expansion on $\ln\alpha$ at $\alpha = 1$,

$$\ln\alpha = \ln 1 - (1-\alpha) - \frac{1}{2}(1-\alpha)^2 - ... < \alpha - 1 - \frac{1}{2}(\alpha - 1)^2$$

Therefore,

$$\begin{aligned} \frac{\partial^2 f}{\partial \alpha^2} &< w[-\frac{1}{\alpha - c/w} + \frac{1}{\alpha(1-\alpha)^2} + \frac{2(\alpha - 1 - \frac{1}{2}(\alpha-1)^2)}{(1-\alpha)^3} + \frac{1}{(1-\alpha)^2}] \\ &= w[-\frac{1}{\alpha - c/w} + \frac{1}{\alpha}] < 0, \text{ for any } \alpha \in (\frac{c}{w}, 1) \end{aligned}$$

Thus, $f(\alpha, c, w)$ is concave in α.

Note that $\lim_{\alpha\to(c/w)^+} f(\alpha, c, w) = w[c/w + \frac{c/w}{1-c/w}\ln(c/w)] < 0$, $\lim_{\alpha\to 1^-} f(\alpha, c, w) = 0$, and $\lim_{\alpha\to 1^-}\frac{\partial f}{\partial \alpha} = -\frac{1}{2} < 0$. Since $f(\alpha, c, w)$ is continuous and concave in α, $f(\alpha, c, w)$ crosses zero only once from below when varying α. That is, there must exist an $\alpha^*(c, w)$ such that when $\alpha \in (\alpha^*(c, w), 1)$, $f(\alpha, c, w) > 0$, which means $E(p_H) > E(p_L)$. Here, $\alpha^*(c, w)$ is defined by $f(\alpha^*, c, w) = 0$, which yields Eq.(3.3).

Appendix C

More Results for Chapter 3

C.1 Heterogeneous Consumer Valuation

In the baseline model, we assumed that consumers have the same willingness-to-pay to avoid unnecessary distraction from the demand factor. Now we relax this assumption by considering heterogeneous consumer valuation and allow consumers' willingness-to-pay to be different from one to another. As we will see, the main results continue to hold.

We assume that consumers' willingness-to-pay satisfies a distribution with a support $[0, v]$. Correspondingly, the demand function is thus $D(p)$, where $p \in [0, v]$ with $D(0) = 1$ and $D(v) = 0$. Technically, we assume that $D(\cdot)$ is twice-continuously differentiable and non-increasing, and the revenue function $pD(p)$ is concave. Denote $p^m = \arg\max_{p\in[0,v]} pD(p)$ as the optimal monopoly price for H. We let $p^m > c$ to rule out trivial cases; otherwise, H's optimal monopoly price is below L's marginal cost, which leads to a simple pure strategy for H regardless of its winning position.

We can conduct a similar analysis as in the baseline model to derive the equilibrium. Again, no pure-strategy equilibrium exists in the second stage price competition. When H wins the first position, as in the baseline model, the two firms randomize their prices over a common support $[\underline{p}, \bar{p}]$ with a possible mass point at the upper bound. The upper bound now is H's optimal monopoly price p^m, since H has no incentive to charge a higher price given the demand function (i.e., $\bar{p} = p^m$). By charging the lower bound price $\underline{p}$, H can attract all consumers and earn the same profit as charging the upper bound price. Therefore, if we define p^l as the solution to $p^l D(p^l) = \alpha p^m D(p^m)$, then $\underline{p} = p^l$. Given the concavity of the revenue function, both p^m and p^l are well defined. As in the baseline model, one exception from the above common-support mixed strategy equilibrium is that when the cost advantage is dominating, H may play a pure strategy (while L still plays a mixed strategy). H is better off playing a pure strategy of simply charging a price equal to L's marginal cost to occupy the entire market, if doing so generates more revenue than randomizing its price; that is, if $cD(c) > \alpha p^m D(p^m)$, or $c > p^l$ by the definition of p^l. Combining the two cases together, the two firms' pricing strategies are characterized by the following cumulative distribution functions:

$$\begin{aligned} F_H(p) &= \begin{cases} 1 - \frac{D(\underline{p})(\underline{p}-c)}{D(p)(p-c)} & p \in [\underline{p}, \bar{p}) \\ 1 & p = \bar{p} \end{cases} \\ F_L(p) &= \begin{cases} \frac{D(p)p - D(\underline{p})\underline{p}}{(1-\alpha)D(p)p} & p \in [\underline{p}, \bar{p}) \\ 1 & p = \bar{p} \end{cases} \end{aligned} \tag{C.1}$$

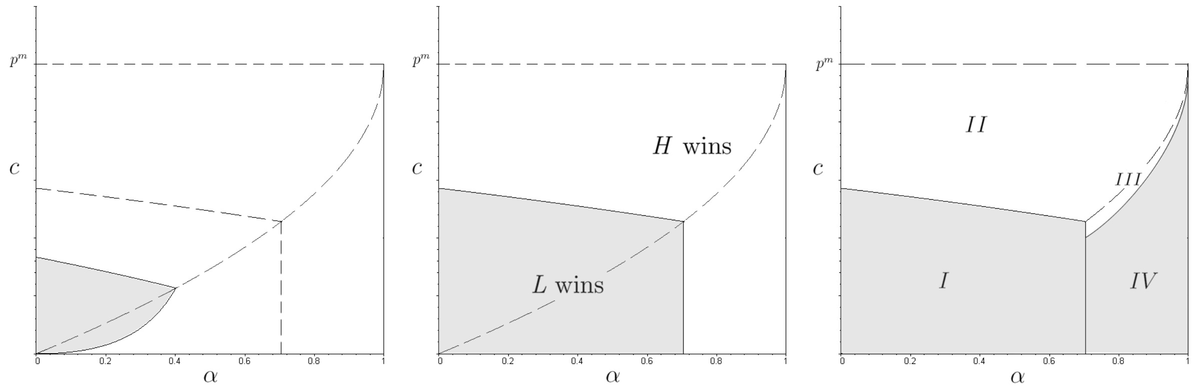
where $\bar{p} = p^m$ and $\underline{p} = \max\{p^l, c\}$. The expected profits are $\pi_H^1 = D(\underline{p})\underline{p}$ and $\pi_L^2 = (1-\alpha)D(\underline{p})(\underline{p} - c)$, respectively.

Similarly, when L wins the first position, we also have a support of the price distribution $[\underline{p}', \bar{p}']$. The upper bound price L may charge is its optimal monopoly price (i.e., $\bar{p}' = \arg\max_{p\in[0,v]} D(p)(p-c)$). The lower bound price is the one that attracts all consumers and leads to the same profit (i.e., $\underline{p}'$ solves $D(\underline{p}')(\underline{p}' - c) = \alpha D(\bar{p}')(\bar{p}' - c)$). The two firms adopt the following strategies:

$$\begin{aligned} F_H(p) &= \frac{D(p)(p-c) - D(\underline{p}')(\underline{p}'-c)}{(1-\alpha)D(p)(p-c)} \qquad p \in [\underline{p}', \bar{p}'] \\ F_L(p) &= \begin{cases} 1 - \frac{D(\underline{p}')\underline{p}'}{D(p)p} & p \in [\underline{p}', \bar{p}') \\ 1 & p = \bar{p}' \end{cases} \end{aligned} \tag{C.2}$$

and the resulting expected profits are $\pi_H^2 = (1-\alpha)D(\underline{p}')\underline{p}'$ and $\pi_L^1 = D(\underline{p}')(\underline{p}' - c)$, respectively.

The equilibrium outcomes (e.g., equilibrium bidding and price dispersion) can be similarly derived as in the baseline model. Figure C.1 illustrates the equilibrium outcome with uniformly distributed consumer valuations (i.e., $D(p) = 1 - \frac{p}{v}$, $p \in [0, v]$). Similar to the baseline case, while the prominent position is always desirable for the low-type firm, the high-type firm may find the less-prominent position more profitable in some cases (shadowed region in Figure C.1(a)). Consequently, the high-type firm bids aggressively to win the prominent position only when either its competitive advantage or the location prominence difference is salient (unshadowed region in Figure C.1(b)); otherwise, the low-type firm wins the prominent position (shadowed region in Figure C.1(b)). Regarding the expected prices from the two locations, a similar pattern is observed: The expected price from the prominent position will be higher (shadowed region in Figure C.1(c)), unless firms' competence difference overrides the location prominence difference (unshadowed region in Figure C.1(c)).



(a) Endogenous Valuation (b) Bidding Outcome (c) Price Dispersion

Figure C.1: Uniformly Distributed Consumer Valuations

Heterogeneity in consumer valuation affects the firms' pricing decision only in that it induces a trade-off between the profit margin and the demand quantity (as in a standard monopoly pricing setting) facing both firms; yet, it does not alter the relative competitive situation between firms and thus has little influence on the major insights we have already obtained. When reflected in the graph, its effect can be depicted as a distortion in shape with no change in pattern.

C.2 Strategic Choice of Ordering: A Brief Analysis

The following analysis mainly focuses on the case with a relatively small portion of sophisticated consumers (i.e., $\beta < \frac{1}{2}$). For the case where $\beta > \frac{1}{2}$, similar analysis applies.

Along the line of backward induction, we start with the second-stage price competition. Notice that in the second stage, sophisticated consumers can only *expect* firms' equilibrium price distributions rather than actually *observe* them. Therefore, by the concept of rational-expectations equilibrium, firms decide their pricing strategies *given* sophisticated consumers' ordering choice (which is in turn rational given firms' equilibrium pricing strategies). It thus implies that a firm's total payment to the auctioneer (i.e., a unit amount times the number of consumer visits) is considered as fixed when it makes the price decision. Therefore, we can derive the equilibrium pricing independent of the first-stage bids.

Again, no pure-strategy equilibrium exists in the second-stage price competition, and firms play mixed strategies in equilibrium. First, we consider the case in which H wins the first position. By Lemma 3.1, firms' equilibrium pricing strategies follow Eq.(3.1), given that all consumers start searching from the first position (i.e., $\sigma = 1$). Notice that $\sigma = 1$ holds in equilibrium if $E(p^1) \leq E(p^2)$. Recall that by Proposition 3.3, when $\alpha < \alpha^*(c, w)$, $E(p^1) < E(p^2)$, where $\alpha^*(c, w)$ is defined by Eq.(3.3). Therefore, when $\alpha < \alpha^*(c, w)$, all consumers starting searching from the first position and both firms pricing according to Eq.(3.1) are an equilibrium in the second stage.

When $\alpha > \alpha^*(c, w)$, however, $E(p^1) > E(p^2)$, and sophisticated consumers thus will not start from the first position. Assume that all sophisticated consumers start sampling from the second position (i.e., $\sigma = 0$). Then, the mixed-strategy equilibrium in pricing can be derived as

$$\begin{array}{ll} F_H^1(p) = \frac{[1-\alpha(1-\beta)](p-\underline{p})}{(1-\alpha)(p-c)} & p \in [\underline{p}, w) \\ F_L^2(p) = \frac{(1-\alpha\beta)(p-\underline{p})}{(1-\alpha)p} & p \in [\underline{p}, w] \end{array} \tag{C.3}$$

where $\underline{p} = \frac{\alpha(1-\beta)}{(1-\alpha\beta)}w$. Notice that H has a mass point at the upper bound as long as $c < \frac{\alpha(1-2\beta)}{1-\alpha\beta}w$. For the above to be an equilibrium, we need to ensure that $E(p^1) \geq E(p^2)$. According to $E(p^1) = E(p^2)$, we can define $\hat{\alpha}(c, w; \beta)$ as

$$\frac{\hat{\alpha}(1-2\beta)}{1-\hat{\alpha}\beta} + \frac{[1-\hat{\alpha}(1-\beta)](\frac{\hat{\alpha}(1-\beta)}{(1-\hat{\alpha}\beta)} - \frac{c}{w})}{1-\hat{\alpha}} \ln \frac{1-\frac{c}{w}}{\frac{\hat{\alpha}(1-\beta)}{(1-\hat{\alpha}\beta)} - \frac{c}{w}} - \frac{\hat{\alpha}(1-\beta)}{1-\hat{\alpha}} \ln \frac{(1-\hat{\alpha}\beta)}{\hat{\alpha}(1-\beta)} = 0 \tag{C.4}$$

Notice that $\alpha^*(c, w)$ in Eq.(3.3) can be rewritten as $\hat{\alpha}(c, w; 0)$. As can be verified, when $\alpha > \hat{\alpha}(c, w; \beta)$, $E(p^1) > E(p^2)$. Therefore, when $\alpha > \hat{\alpha}(c, w; \beta)$, sophisticated consumers starting from the second position and firms pricing according to Eq.(C.3) are an equilibrium in the second stage.

In the region where $\hat{\alpha}(c, w; 0) < \alpha < \hat{\alpha}(c, w; \beta)$, as can be concluded from the above analysis, there exists no equilibrium in which sophisticated consumers play pure strategy. Instead, sophisticated consumers play mixed strategy with $0 < \sigma < 1$, and the expected prices from the two positions are equal (i.e., $E(p^1) = E(p^2)$). For any $\beta' \in (0, \beta)$, we can define a curve $\alpha = \hat{\alpha}(c, w; \beta')$ according to Eq.(C.4). When $(\alpha, \frac{c}{w})$ lies on that curve, sophisticated consumers playing mixed strategy $\sigma = 1 - \frac{\beta'}{\beta}$, and firms pricing the following strategies are an equilibrium in the second stage:

$$\begin{array}{ll} F_H^1(p) = \frac{[1-\alpha(1-\beta')](p-\underline{p})}{(1-\alpha)(p-c)} & p \in [\underline{p}, w) \\ F_L^2(p) = \frac{(1-\alpha\beta')(p-\underline{p})}{(1-\alpha)p} & p \in [\underline{p}, w] \end{array} \tag{C.5}$$

where $\underline{p} = \frac{\alpha(1-\beta')}{(1-\alpha\beta')}w$.

We next consider the case where L wins the first position. Proposition 3.3 indicates $E(p_L^1) > E(p_H^2)$, given that all consumers start searching from the first position. Therefore, $\sigma = 1$ cannot be part of an equilibrium. We then assume that all sophisticated consumers start sampling from the second position ($\sigma = 0$). The resulting pricing strategies are

$$\begin{array}{ll} F_H^2(p) = \frac{(1-\alpha\beta)(p-\underline{p})}{(1-\alpha)(p-c)} & p \in [\underline{p}, w] \\ F_L^1(p) = \frac{[1-\alpha(1-\beta)](p-\underline{p})}{(1-\alpha)p} & p \in [\underline{p}, w) \end{array} \tag{C.6}$$

where $\underline{p} = \frac{\alpha(1-\beta)(w-c)}{1-\alpha\beta} + c$. As we can show, F_L first order stochastically dominates F_H such that $E(p_L^1) > E(p_H^2)$, which is consistent with $\sigma = 0$. Thus, in the second-stage game when L wins the first position, the equilibrium is that sophisticated consumers start from the second position and the two firms price according to Eq.(C.6).

We can summarize firms' equilibrium expected profits from sales in different positions within different parameter regions as follows:

$$
\begin{aligned}
(\pi_H^1, \pi_L^2) &= \begin{cases} (c, 0) & \text{if } 0 < \alpha < \frac{c}{w}; \\ (\alpha w, (1-\alpha)(\alpha w - c)) & \text{if } \frac{c}{w} < \alpha < \hat{\alpha}(c, w; 0); \\ \Big(\alpha(1-\beta)w, [1-\alpha(1-\beta)](\frac{\alpha(1-\beta)}{(1-\alpha\beta)}w - c)\Big) & \text{if } \hat{\alpha}(c, w; \beta) < \alpha < 1; \\ \Big(\alpha(1-\beta')w, [1-\alpha(1-\beta')](\frac{\alpha(1-\beta')}{(1-\alpha\beta')}w - c)\Big) & \text{if } \alpha = \hat{\alpha}(c, w; \beta'), \forall \beta' \in (0, \beta). \end{cases} \\
(\pi_H^2, \pi_L^1) &= \Big([1-\alpha(1-\beta)](\tfrac{\alpha(1-\beta)(w-c)}{1-\alpha\beta} + c), \alpha(1-\beta)(w-c)\Big)
\end{aligned}
\tag{C.7}
$$

For the first-stage bidding, notice that here the weighting factors (or the expected clicks at the first position) may not equal 1. For example, in the parameter region III in Figure 3.5(c), $\omega_H = 1$ and $\omega_L = 1 - \alpha\beta$ because when L wins the first position, its equilibrium price expectation is higher so that sophisticated non-shoppers will visit the second position directly. Nevertheless, as is shown in the paper, firms' weakly dominant strategy is to submit a per-click bid $b_i = \frac{\max\{\Delta\pi_i, 0\}}{\omega_i}$, where $\Delta\pi_i = \pi_i^1 - \pi_i^2$ and $i \in \{H, L\}$, and the auctioneer ranks the firms according to the score $s_i = \omega_i b_i = \max\{\Delta\pi_i, 0\}$, which is independent of ω_i.

By comparing $\Delta\pi_H$ and $\Delta\pi_L$ (and then checking whether the winning bids are positive), we can derive the bidding outcome and pin down the boundaries of firms' winning regions. Use Figure 3.5(c) for illustration. When $0 < \alpha < \frac{c}{w}$, $\Delta\pi_H > \Delta\pi_H$ if $\frac{c}{w} > \frac{2-\alpha}{3-\alpha-\alpha\beta}$ (region II). When $\frac{c}{w} < \alpha < \hat{\alpha}(c, w; 0)$, comparing $\Delta\pi_H$ and $\Delta\pi_L$ leads to the boundary of region III: $\frac{c}{w} = \frac{\alpha\beta(1-\alpha)(2-\alpha)}{\alpha^2(1+\beta-\beta^2)-\alpha(4-\beta)+2}$. Note that the three curves $\frac{c}{w} = \frac{2-\alpha}{3-\alpha-\alpha\beta}$, $\frac{c}{w} = \frac{\alpha\beta(1-\alpha)(2-\alpha)}{\alpha^2(1+\beta-\beta^2)-\alpha(4-\beta)+2}$, and $\frac{c}{w} = \alpha$ intersect at one point. When $\hat{\alpha}(c, w; \beta) < \alpha < 1$, $\Delta\pi_H > \Delta\pi_H$ if $\alpha > \frac{(2-\beta)-\sqrt{5\beta^2-6\beta+2}}{(1-\beta)(1+2\beta)}$ (region IV). When $\hat{\alpha}(c, w; 0) < \alpha < \hat{\alpha}(c, w; \beta)$, for any curve $\alpha = \hat{\alpha}(c, w; \beta')$ (with $\beta' \in (0, \beta)$) in between, its intersection with $\frac{c}{w} = \frac{\alpha(2-\alpha)(1-\alpha)(\beta-\beta')}{(1-\alpha\beta')[(1+\beta-\beta^2-\beta\beta')\alpha^2-(4-\beta-\beta')\alpha+2]}$ defines a cutoff in comparing $\Delta\pi_H$ and $\Delta\pi_L$. The boundary of region V consists of all these cutoffs. Together with the discussion on the expected prices in the second-stage pricing game, the spatial price dispersion pattern can be summarized as by Figure 3.5(c).

C.3 Equilibrium Pricing in the Three-Firms Case

In this section, we provide a complete description of the equilibrium pricing strategies in the case of three competing firms. We organize the results by different parameter regions under different scenarios. The proofs to the first two results are outlined, and the rest can be analyzed in a similar fashion. For convenience, denote $H-L-L$ as the case when H stays in the first position and two L firms are in the second and third positions. Similar interpretations apply to $L-H-L$ and $L-L-H$. Also, we call the firm in position i as firm i, where $i=1,2,3$. As before, we use $F_i(\cdot)$ to represent the cumulative distribution function (*cdf*) for the pricing strategy of firm i.

(1) The Case of $H-L-L$

Result C.1. *In the case of $H-L-L$, when $\frac{c}{w}<\frac{\alpha^2}{1+\alpha(1-\alpha)}$, the equilibrium pricing strategies are:*

$$\begin{aligned}
F_1(p) &= \begin{cases} 1-\frac{\bar{p}_2-c}{p-c} & p\in[\bar{p}_2,\bar{p}_1) \\ 1 & p=\bar{p}_1 \end{cases} \\
F_2(p) &= \begin{cases} 1-\frac{\bar{p}_3-c}{p-c} & p\in[\bar{p}_3,\bar{p}_2) \\ 1-\frac{(\bar{p}_1-p)}{(1-\alpha)p} & p\in[\bar{p}_2,\bar{p}_1] \end{cases}, \\
F_3(p) &= \quad 1-\frac{\alpha(\bar{p}_2-p)}{(1-\alpha)(p-c)} \quad p\in[\bar{p}_3,\bar{p}_2],
\end{aligned} \tag{C.8}$$

where $\bar{p}_1=w$, $\bar{p}_2=\frac{1}{1+\alpha(1-\alpha)}\bar{p}_1$, and $\bar{p}_3=\alpha(\bar{p}_2-c)+c$.

For convenience, we use the superscript "+" to denote the cdf in the upper half of the support, and we use the superscript "−" as the cdf in the lower half of the support. For example, in Eq.C.8, we refer to $F_2(p)=1-\frac{(\bar{p}_1-p)}{(1-\alpha)p}$ for $p\in[\bar{p}_2,\bar{p}_1]$ as $F_2^+(p)$ and refer to $F_2(p)=1-\frac{\bar{p}_3-c}{p-c}$ for $p\in[\bar{p}_3,\bar{p}_2)$ as $F_2^-(p)$.

First, we show that $F_i(\cdot)$ $(i=1,2,3)$ are well-defined cumulative distribution functions. Notice that $c<\bar{p}_3<\bar{p}_2<\bar{p}_1=w$, so that the supports are well defined. Also, we can check all the bounds: $F_1(\bar{p}_2)=0$, $F_1^-(\bar{p}_1)=1-\frac{\bar{p}_2-c}{\bar{p}_1-c}<1$, $F_2(\bar{p}_3)=0$, and $F_2^-(\bar{p}_2)=F_2^+(\bar{p}_2)$ because $\frac{\bar{p}_3-c}{\bar{p}_2-c}=\frac{(\bar{p}_1-\bar{p}_2)}{(1-\alpha)\bar{p}_2}=\alpha$, $F_2(\bar{p}_1)=1$, $F_3(\bar{p}_3)=1-\frac{\alpha(\bar{p}_2-\bar{p}_3)}{(1-\alpha)(\bar{p}_3-c)}=$

$1-\frac{\alpha(1-\alpha)(\bar{p}_2-c)}{(1-\alpha)(\bar{p}_3-c)}=0$, and $F_3(\bar{p}_2)=1$. Moreover, all $F_i(p)$ are increasing in p. Therefore, all $F_i(\cdot)$ are well-defined *cdf*s.

Second, we can show that each position i yields a constant expected profit π_i within the entire support, $i=1,2,3$. For example, consider the second position. For $p\in[\bar{p}_3,\bar{p}_2)$,

$$\pi_2^-(p)=\alpha(1-\alpha)(p-c)+(1-\alpha)^2[1-F_3(p)](p-c),$$

where the first part on the right-hand side accounts for the demand from those consumers who stop searching at the second position (they purchase from position 2 for sure because p is lower than the lower bound of F_1's support, $\bar{p}_2$), and the second part accounts for the demand from shoppers when its price is lower than the third position. Substituting in $F_3(p)$, we have:

$$\begin{aligned}\pi_2^-(p) &= \alpha(1-\alpha)(p-c)+(1-\alpha)^2\frac{\alpha(\bar{p}_2-p)}{(1-\alpha)(p-c)}(p-c)\\ &\equiv \alpha(1-\alpha)(\bar{p}_2-c).\end{aligned}$$

Similarly, for $p\in[\bar{p}_2,\bar{p}_1]$, we have:

$$\begin{aligned}\pi_2^+(p) &= \alpha(1-\alpha)[1-F_1(p)](p-c)\\ &= \alpha(1-\alpha)\frac{\bar{p}_2-c}{p-c}(p-c)\equiv\alpha(1-\alpha)(\bar{p}_2-c).\end{aligned}$$

Therefore, firm 2 achieves a constant expected profit level $\pi_2=\alpha(1-\alpha)(\bar{p}_2-c)$ when charging any price within its support $[\bar{p}_3,\bar{p}_1]$. Similar analysis applies for both $\pi_1=\alpha\bar{p}_1$ and $\pi_3=(1-\alpha)^2(\bar{p}_3-c)$.

Finally, we need to verify that there is no profitable unilateral deviation. For example, if H deviates to price $p\in[\bar{p}_3,\bar{p}_2)$, the profit function would then become:

$$\begin{aligned}\pi_1^{'}(p) &= \alpha p+\alpha(1-\alpha)p\left[1-F_2^-(p)\right]+(1-\alpha)^2p\left[1-F_2^-(p)\right][1-F_3(p)]\\ &= \alpha p+\frac{\bar{p}_3-c}{p-c}\alpha(1-\alpha)(\bar{p}_2-c)\frac{p}{p-c},\end{aligned}$$

which is convex in p. Notice that when $\frac{c}{w}<\frac{\alpha^2}{1+\alpha(1-\alpha)}$, $\pi_1(\bar{p}_2)\geq\pi_1^{'}(\bar{p}_3)$. Therefore, $\pi_1^{'}(p)<\pi_1(\bar{p}_2)$ for $\forall p\in[\bar{p}_3,\bar{p}_2)$. Thus, when $\frac{c}{w}<\frac{\alpha^2}{1+\alpha(1-\alpha)}$, H will not deviate to charge a price below the given support. Similarly, we can show that it is not profitable for firm 3 to charge a price above its given support.

Altogether, we can conclude that the price strategy described is a mixed-strategy equilibrium.

Result C.2. *In the case of $H-L-L$, when $\frac{\alpha^2}{1+\alpha(1-\alpha)} < \frac{c}{w} < \min\{\alpha, \frac{1-\alpha(1-\alpha)}{[1+\alpha(1-\alpha)]^2}\}$, the equilibrium pricing strategies are:*

$$\begin{aligned}
F_1(p) &= \begin{cases} F_1^*(p) & p \in [\bar{p}_4, \bar{p}_3] \\ 1-\frac{\alpha\bar{p}_1 - c}{\alpha(p-c)} & p \in [\bar{p}_2, \bar{p}_1) \\ 1 & p = \bar{p}_1 \end{cases} \\
F_2(p) &= \begin{cases} F_2^*(p) & p \in [\bar{p}_4, \bar{p}_3) \\ 1-\frac{\alpha(\bar{p}_2 - c)}{p-c} & p \in [\bar{p}_3, \bar{p}_2) \\ 1-\frac{(\bar{p}_1-p)}{(1-\alpha)p} & p \in [\bar{p}_2, \bar{p}_1] \end{cases} \\
F_3(p) &= \begin{cases} F_3^*(p) & p \in [\bar{p}_4, \bar{p}_3) \\ 1-\frac{\alpha(\bar{p}_2-p)}{(1-\alpha)(p-c)} & p \in [\bar{p}_3, \bar{p}_2], \end{cases}
\end{aligned} \tag{C.9}$$

where $\bar{p}_1 = w$, $\bar{p}_2 = \frac{1}{1+\alpha(1-\alpha)}w$, $\bar{p}_4 = \alpha w$, and $\bar{p}_3$ is the solution between $\bar{p}_4$ and $\bar{p}_2$ to the equation:

$$[1+\alpha(1-\alpha)]\,\bar{p}_3^2 - [\alpha(1-\alpha)(1+2c)+2c]\,\bar{p}_3 + [1+\alpha(1-\alpha)]^2 c^2 = 0, \tag{C.10}$$

and $\{F_1^*(p), F_2^*(p), F_3^*(p)\}$ solves:

$$\begin{cases} \alpha p + \alpha(1-\alpha)p[1-F_2^*(p)] + (1-\alpha)^2 p[1-F_2^*(p)][1-F_3^*(p)] = \alpha w \\ \alpha(1-\alpha)(p-c)[1-F_1^*(p)] + (1-\alpha)^2(p-c)[1-F_1^*(p)][1-F_3^*(p)] = (1-\alpha)(\alpha w - c) \\ (1-\alpha)^2(p-c)[1-F_1^*(p)][1-F_2^*(p)] = (1-\alpha)^2(\alpha w - c). \end{cases} \tag{C.11}$$

Following similar analysis, we can show that the strategy is indeed an equilibrium, as is briefly outlined below.

First, the supports and the *cdf*s are well defined. Notice that when $\frac{\alpha^2}{1+\alpha(1-\alpha)} < \frac{c}{w}$, $F_1(\bar{p}_2) = 1 - \frac{\alpha\bar{p}_1 - c}{\alpha(\bar{p}_2 - c)} > 0$, which indicates that H may charge a price lower than $\bar{p}_2$ with positive probability. Also, notice that the parameter region $\frac{\alpha^2}{1+\alpha(1-\alpha)} < \frac{c}{w} < \min\{\alpha, \frac{1-\alpha(1-\alpha)}{[1+\alpha(1-\alpha)]^2}\}$ ensures that there is a unique solution to Eq.(C.10) between $\bar{p}_4$ and $\bar{p}_2$.

In fact, if we denote the right-hand side of Eq.(C.10) as $g(\bar{p}_3)$, under the given parameter condition, $g(\bar{p}_4) < 0$ and $g(\bar{p}_2) > 0$, which ensures that $\bar{p}_3$ is well defined. Moreover, notice that $\bar{p}_3$ actually solves $F_1^*(\bar{p}_3) = F_1(\bar{p}_2)$. To see this, by substituting $p = \bar{p}_3$ and $F_1^*(\bar{p}_3) = 1 - \frac{\alpha\bar{p}_1 - c}{\alpha(\bar{p}_2 - c)}$ into Eq.(C.11), we can then solve the last two equations of Eq.(C.11) together and get:

$$\begin{cases} F_2^*(\bar{p}_3) = 1 - \frac{\alpha(\bar{p}_2 - c)}{\bar{p}_3 - c} \\ F_3^*(\bar{p}_3) = 1 - \frac{\alpha(\bar{p}_2 - \bar{p}_3)}{(1-\alpha)(\bar{p}_3 - c)}. \end{cases}$$

Substituting back into the first equation of Eq.(C.11), we have Eq.(C.10).

Second, firms achieve constant profit within their price supports: (i) Notice that the left-hand sides in Eq.(C.11) are in fact the profit functions for the three firms when charging $p \in [\bar{p}_4, \bar{p}_3]$, and the right-hand sides are the constant expected profit they achieve over their entire price supports. Therefore, Eq.(C.11) ensures that the three firms all achieve a constant profit level when pricing $p \in [\bar{p}_4, \bar{p}_3]$. (ii) $F_1(\bar{p}_3) = F_1(\bar{p}_2)$ indicates that F_1 does not put any mass over the interval $(\bar{p}_3, \bar{p}_2)$, which means that H does not charge any price between $\bar{p}_3$ and $\bar{p}_2$. Thus, for $p \in (\bar{p}_3, \bar{p}_2)$, firm 2 and firm 3 achieve a constant profit:

$$\begin{cases} \pi_2(p) = \alpha(1-\alpha)(p-c)[1 - F_1(\bar{p}_2)] + (1-\alpha)^2 (p-c)[1 - F_1(\bar{p}_2)][1 - F_3(p)] \equiv (1-\alpha)(\alpha w - c) \\ \pi_3(p) = (1-\alpha)^2 (p-c)[1 - F_1(\bar{p}_2)][1 - F_2^*(p)] \equiv (1-\alpha)^2(\alpha w - c). \end{cases}$$

(iii) $F_3(\bar{p}_2) = 1$, indicating that firm 3 does not charge any price above $\bar{p}_2$. Therefore, for $p \in (\bar{p}_2, \bar{p}_1)$,

$$\begin{cases} \pi_1(p) = \alpha p + \alpha(1-\alpha) p [1 - F_2(p)] \equiv \alpha w \\ \pi_2(p) = \alpha(1-\alpha)(p-c)[1 - F_1(p)] \equiv (1-\alpha)(\alpha w - c). \end{cases}$$

Finally, as we can check, firm 1 pricing $p \in (\bar{p}_3, \bar{p}_2)$ and firm 3 pricing $p \in (\bar{p}_2, \bar{p}_1)$ both result in lower profits. Therefore, there is no profitable deviation. As a result, the pricing strategy described is a mixed-strategy equilibrium.

Result C.3. *In the case of $H - L - L$, when $\min\{\alpha, \frac{1-\alpha(1-\alpha)}{[1+\alpha(1-\alpha)]^2}\} < \frac{c}{w} < \alpha$, the equilibrium* *pricing strategies are:*

$$\begin{aligned} F_1(p) &= \begin{cases} F_1^*(p) & p \in [\bar{p}_3, \bar{p}_2) \\ 1 - \frac{\alpha \bar{p}_1 - c}{\alpha(p-c)} & p \in [\bar{p}_2, \bar{p}_1) \\ 1 & p = \bar{p}_1 \end{cases} \\ F_2(p) &= \begin{cases} F_2^*(p) & p \in [\bar{p}_3, \bar{p}_2) \\ 1 - \frac{(\bar{p}_1 - p)}{(1-\alpha)p} & p \in [\bar{p}_2, \bar{p}_1] \end{cases} \\ F_3(p) &= \quad F_3^*(p) \quad p \in [\bar{p}_3, \bar{p}_2], \end{aligned} \tag{C.12}$$

where $\bar{p}_1 = w$, $\bar{p}_2 = \frac{1}{1+\alpha(1-\alpha)}w$, *and* $\bar{p}_3 = \alpha w$, *and* $\{F_1^*(p), F_2^*(p), F_3^*(p)\}$ *solves Eq.*(C.11).

The analysis in this case is similar to that for Result 2.

Result C.4. *In the case of* $H - L - L$, *when* $\frac{c}{w} > \alpha$, *firm 1 pricing* $p_1 = c$, *firm 3 pricing* $p_3 = w$, *and firm 2 pricing according to:*

$$F_2(p) = \begin{cases} 1 - \frac{c - \alpha p}{(1-\alpha)p} & p \in [c, w) \\ 1 & p = w \end{cases}, \tag{C.13}$$

is an equilibrium.

This is a trivial case in which the cost advantage is so significant that H simply charges c and both L firms achieve zero profit. Firm 2 prices in a way that firm 1 has no profitable deviation.

(2) The Case of $L - H - L$

Result C.5. *In the case of* $L - H - L$, *when* $\frac{c}{w} < \alpha$, *the equilibrium pricing strategies are:*

$$\begin{aligned} F_1(p) &= \begin{cases} 1 - \frac{\bar{p}_2}{p} & p \in [\bar{p}_2, \bar{p}_1) \\ 1 & p = \bar{p}_1 \end{cases} \\ F_2(p) &= \begin{cases} 1 - \frac{\bar{p}_3 - c}{p - c} & p \in [\bar{p}_3, \bar{p}_2) \\ 1 - \frac{(\bar{p}_1 - p)}{(1-\alpha)(p-c)} & p \in [\bar{p}_2, \bar{p}_1] \end{cases} \\ F_3(p) &= \quad 1 - \frac{\alpha(\bar{p}_2 - p)}{(1-\alpha)p} \quad p \in [\bar{p}_3, \bar{p}_2], \end{aligned} \tag{C.14}$$

where $\bar{p}_1 = w$, $\bar{p}_2 = \frac{\bar{p}_1 + (1-\alpha)c}{1+\alpha(1-\alpha)}$, *and* $\bar{p}_3 = \alpha_2 \bar{p}_2$.

This case can be analyzed similarly to the analysis of Result 1.

Result C.6. *In the case of $L-H-L$, when $\frac{c}{w} > \alpha$, firm 1 pricing $p_1 = w$, firm 2 pricing $p_2 = c$, and firm 3 pricing according to:*

$$F_3(p) = \begin{cases} 1 - \frac{c-\alpha p}{(1-\alpha)p} & p \in [c, w) \\ 1 & p = w \end{cases}, \tag{C.15}$$

is an equilibrium.

This is a trivial case, similar to Result 4.

(3) The Case of $L-L-H$

Result C.7. *In the case of $L-L-H$, the equilibrium pricing strategies are:*

$$\begin{aligned} F_1(p) &= \begin{cases} 1 - \frac{\bar{p}_2 - c}{p-c} & p \in [\bar{p}_2, \bar{p}_1) \\ 1 & p = \bar{p}_1 \end{cases} \\ F_2(p) &= \begin{cases} 1 - \frac{\bar{p}_3}{p} & p \in [\bar{p}_3, \bar{p}_2) \\ 1 - \frac{(\bar{p}_1 - p)}{(1-\alpha)(p-c)} & p \in [\bar{p}_2, \bar{p}_1] \end{cases} \\ F_3(p) &= \quad 1 - \frac{\alpha(\bar{p}_2 - p)}{(1-\alpha)(p-c)} \quad p \in [\bar{p}_3, \bar{p}_2] \end{aligned}, \tag{C.16}$$

where $\bar{p}_1 = w$, $\bar{p}_3 = \alpha(\bar{p}_2 - c) + c$, and

$$\bar{p}_2 = \frac{-(1-\alpha)(1-2\alpha)c + w + \sqrt{5(1-\alpha)^2 c^2 - 2(1-\alpha)(1-2\alpha)wc + w^2}}{2[1+\alpha(1-\alpha)]}.$$

This case can be analyzed similarly to the analysis of Result 1 and Result 5.

C.4 More Results for the Three-Firms Case

Recall the equilibrium profits in the price competition from Table 3.2. We denote the three firms' second-stage equilibrium profits in the case of $H-L-L$ (i.e., H wins the first position and the two L firms are in the second and third positions) as u_H^1, u_L^2, and u_L^3, respectively. Similarly, we denote the equilibrium profits in the $L-H-L$ case as v_L^1, v_H^2, and v_L^3, and we denote the equilibrium profits in the $L-L-H$ case as w_L^1, w_L^2, and w_H^3.

In deriving the bidding outcomes, we focus on the particular type of equilibria in which the two low-type firms behave symmetrically, that is, they adopt the same bidding strategies. Figure C.2 summarizes the bidding outcome. In the shadowed region, which corresponds to the condition that $u_H^1 - w_H^3 < \min\{\frac{w_L^2 - v_L^3}{1-\alpha}, \frac{w_L^1 - w_L^2}{\alpha}\}$, H bids lower than the two L firms and stays in the third position. More specifically, in this region, $b_H = u_H^1 - w_H^3$ and $b_L = (1-\alpha)(u_H^1 - w_H^3) + w_L^1 - w_L^2$. Notice that the two low-type firms submit the same bid b_L because they are indifferent between one position higher or not, and hence win a higher position with equal probability. Also, notice that in our setting, all three firms generate the same number of clicks when they are in the same position. Therefore, they have the same weighting factor, and the ranking is determined by the amount of their per-click bids b_i. As a result, when $u_H^1 - w_H^3 < \frac{w_L^1 - w_L^2}{\alpha}$, $b_H < b_L$, and H is therefore placed in the third position. To ensure that the bidding strategy is an equilibrium, we need to further check the possible deviations. As we can see, given b_L, H does not want to outbid L because its net profit would be $u_H^1 - b_L$, which is lower than its net profit in the third position, w_H^3. Similarly, given b_H and the other L firm's bid b_L, neither L firm has a profitable deviation: First, staying in the first position yields a net profit $w_L^1 - b_L$, which equals the net profit from the second position $w_L^2 - (1-\alpha)\, b_H$ (notice that the number of click-throughs in the second position is $1-\alpha$). Second, underbidding to stay in the third position is not optimal either because the net profit then would be v_L^3—lower than the in-equilibrium net profit $w_L^2 - (1-\alpha)\, b_H$ as $b_H < \frac{w_L^2 - v_L^3}{1-\alpha}$.

On the other hand, in the unshadowed region in Figure C.2 (i.e., when $u_H^1 - w_H^3 > \max\{v_L^1 - u_L^3, \frac{u_L^2 - u_L^3}{1-\alpha}\}$), H outbids both L firms and wins the first position. In this case, $b_H = u_H^1 - w_H^3$, and $b_L = \frac{u_L^2 - u_L^3}{1-\alpha}$. Following similar arguments, we can show that none of the three firms has a profitable deviation. For H, because $b_L < u_H^1 - w_H^3$, underbidding to be in the third position would lead to a lower net profit (w_H^3) than being in the first position ($u_H^1 - b_L$). For L, staying in the second or the third position results in the same net profit because $u_L^2 - (1-\alpha)\, b_L = u_L^3$. It is not profitable for L to outbid H because being in the first position yields a payoff of $v_L^1 - b_H$, which is lower than the payoff in the second or third position (u_L^3).

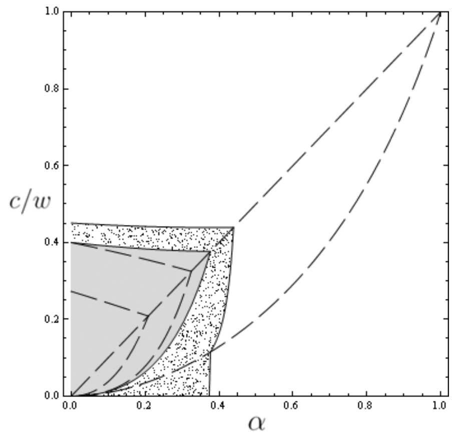


Figure C.2: Bidding Outcome in the Case of Three Firms

An interesting result is found in the dotted region in Figure C.2. In this region, firms may adopt mixed-strategy bidding in equilibrium. We use an example to better illustrate the idea.

Example C.1. *When $\alpha = 0.4$, $c = 0.3$, and $w = 1$, in equilibrium, both L firms symmetrically bid $b_L = 0.219$ and H adopts mixed-strategy bidding. With probability $p = 0.23$, H bids as high as $b_H^{'} = 0.352$ and wins the first position; with probability $1 - p = 0.77$, H bids as low as $b_H = 0.155$ and stays in the third position.*

The example illustrates a bidding outcome in which the high-type firm switches between a top position and a lower position. In this case, $b_L = u_H^1 - w_H^3$ so that H is indifferent between attaining the first position and attaining the third one. For this reason, H is willing to mix its bid. H mixes in such a way that neither L wants to overbid or underbid its counterpart (i.e., bidding b_L is optimal, given that the other L firm also bids b_L). For this reason, the mixing probability p satisfies:

$$p = \frac{b_L - (1-\alpha)\, b_H - w_L^1 + w_L^2}{(u_L^2 - u_L^3) - (w_L^1 - w_L^2) - (1-\alpha)\, b_H + \alpha b_L}.$$

In addition, H's high bid $b_H^{'} = (1-\alpha)\, b_L + v_L^1 - u_L^2$ is high enough that L will not deviate to bid higher than $b_H^{'}$; meanwhile, H's low bid $b_H = \frac{w_L^2 - v_L^3}{1-\alpha}$ is low enough that bidding lower than b_H is not optimal for L either. As a result, $b_H < b_L < b_H^{'}$ in equilibrium.

As we can see from Figure C.2, the dotted region with mixed-strategy bidding serves as a natural transition between the two deterministic cases that involves only pure-strategy bidding.

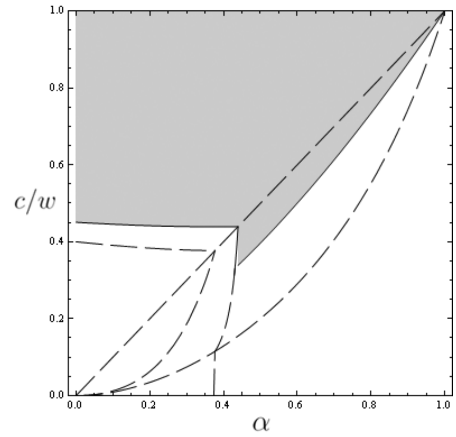


Figure C.3: Price Dispersion in the Case of Three Firms

Figure C.3 illustrates the spatial price dispersion pattern in the three-firms case. As we can see, similar results exist. In the highlighted region, the high-type firm wins the first position, and the expected price from the first position is lower than that from the second position, indicating that, depending on the endogenous competitive situation, an expensive location may not necessarily be associated with expensive products.

Appendix D

Proofs for Chapter 4

Proof of Lemma 4.1. According to Table 4.1, (i) If $\frac{1}{3} \leq \psi < 1$, then $\frac{\partial^2}{\partial\psi\partial\theta}\pi_M^*(\psi,\theta) = -1 < 0$ and $\frac{\partial^2}{\partial\psi\partial\theta}\pi_N^*(\psi,\theta) = -\frac{1}{4(1-\theta)^2} < 0$.

(ii) If $0 < \psi < \frac{1}{3}$, then $\frac{\partial^2}{\partial\psi\partial\theta}\pi_M^*(\psi,\theta) = \frac{f(\psi,\theta)}{(3-3\theta-3\psi+7\theta\psi)^4}$, where $f(\psi,\theta)$ is a polynomial function of ψ and θ with degree of 8 ($\psi^4\theta^4$). We can show that $f(\psi,\theta)$ is convex in ψ for $0 < \psi < \frac{1}{3}$ and $0 < \theta < \frac{1}{2}$. We can verify that $f(0,\theta) < 0$ and $f(\frac{1}{3},\theta) < 0$ for $\forall\theta \in (0,\frac{1}{2})$. Therefore, $f(\psi,\theta) < 0$ for $\forall\psi \in (0,\frac{1}{3})$ and $\forall\theta \in (0,\frac{1}{2})$, which implies $\frac{\partial^2}{\partial\psi\partial\theta}\pi_M^*(\psi,\theta) < 0$. Similarly, $\frac{\partial^2}{\partial\psi\partial\theta}\pi_N^*(\psi,\theta) = \frac{g(\psi,\theta)}{(1-\theta)^2(3-3\theta-3\psi+7\theta\psi)^4}$, where $g(\psi,\theta)$ is a polynomial function of ψ and θ with degree of 10 ($\psi^4\theta^6$). Again, we can show that $g(\psi,\theta)$ is convex in ψ for $0 < \psi < \frac{1}{3}$ and $0 < \theta < \frac{1}{2}$. Since $g(0,\theta) < 0$ and $g(\frac{1}{3},\theta) < 0$ for $\forall\theta \in (0,\frac{1}{2})$, we can conclude that $g(\psi,\theta) < 0$ for $\forall\psi \in (0,\frac{1}{3})$ and $\forall\theta \in (0,\frac{1}{2})$. Therefore, $\frac{\partial^2}{\partial\psi\partial\theta}\pi_N^*(\psi,\theta) < 0$.

Proof of Proposition 4.1. Define $h(\psi,\theta) = \pi_M^*(\psi,\theta) + \pi_N^*(\psi,\theta)$. Since both $\pi_M^*(\psi,\theta)$ and $\pi_N^*(\psi,\theta)$ are submodular by Lemma 4.1, $h(\psi,\theta)$ is also submodular. We then define

$$\begin{aligned} H(\theta,\psi_1,\psi_2) &\equiv [\pi_M^*(\psi_2,\theta) - \pi_M^*(\psi_1,\theta)] - [\pi_N^*(\psi_1,\theta) - \pi_N^*(\psi_2,\theta)] \\ &= h(\psi_2,\theta) - h(\psi_1,\theta). \end{aligned}$$

If $0 < \psi_1 < \psi_2 < \frac{1}{3}$ or $\frac{1}{3} \leq \psi_1 < \psi_2 < 1$, then

$$\frac{\partial}{\partial\theta}H(\theta,\psi_1,\psi_2) = \frac{\partial}{\partial\theta}h(\psi_2,\theta) - \frac{\partial}{\partial\theta}h(\psi_1,\theta) = \frac{\partial^2}{\partial\theta\partial\psi}h\left(\bar{\psi},\theta\right)(\psi_2-\psi_1)$$

where the second equality is by applying the Mean Value Theorem and $\bar{\psi} \in [\psi_1,\psi_2]$. Since $\frac{\partial^2}{\partial\theta\partial\psi}h\left(\bar{\psi},\theta\right) < 0$, $\frac{\partial}{\partial\theta}H(\theta,\psi_1,\psi_2) < 0$.

If $0<\psi_1<\frac{1}{3}\leq\psi_2<1$, then

$$\begin{aligned}\frac{\partial}{\partial\theta}H\left(\theta,\psi_1,\psi_2\right) &= \frac{\partial}{\partial\theta}h\left(\psi_2,\theta\right)-\frac{\partial}{\partial\theta}h\left(\psi_1,\theta\right)\\ &= \int_{\psi_1}^{\frac{1}{3}}\frac{\partial^2}{\partial\theta\partial\psi}h\left(\psi,\theta\right)d\psi+\int_{\frac{1}{3}}^{\psi_2}\frac{\partial^2}{\partial\theta\partial\psi}h\left(\psi,\theta\right)d\psi.\end{aligned}$$

Since $\frac{\partial^2}{\partial\theta\partial\psi}h\left(\psi,\theta\right)<0$, $\frac{\partial}{\partial\theta}H\left(\theta,\psi_1,\psi_2\right)<0$.

In sum, $H\left(\theta,\psi_1,\psi_2\right)$ is decreasing in θ. Notice $H\left(0,\psi_1,\psi_2\right)=\frac{5(\psi_2-\psi_1)}{9(1-\psi_1)(1-\psi_2)}$ (in the case of $0<\psi_1<\psi_2<\frac{1}{3}$), or $\frac{\psi_2-\psi_1}{4}$ (in the case of $\frac{1}{3}\leq\psi_1<\psi_2<1$), or $\frac{20-9(1-\psi_2)(3+\psi_1)}{36(1-\psi_2)}$ (in the case of $0<\psi_1<\frac{1}{3}\leq\psi_2<1$). We can verify that $H\left(0,\psi_1,\psi_2\right)>0$ in all three cases. Similarly, $H\left(\frac{1}{2},\psi_1,\psi_2\right)=-\frac{\psi_2-\psi_1}{2}<0$ in all three cases. Since $H\left(\theta,\psi_1,\psi_2\right)$ is continuous in θ, we can conclude that there exists a cutoff $\theta^*\in(0,\frac{1}{2})$ such that $H\left(\theta^*,\psi_1,\psi_2\right)=0$, $H\left(\theta,\psi_1,\psi_2\right)>0$ for $\theta<\theta^*$, and $H\left(\theta,\psi_1,\psi_2\right)<0$ for $\theta>\theta^*$. Notice that $\pi_M^*\left(\psi_2,\theta\right)-\pi_M^*\left(\psi_1,\theta\right)>0$ always holds, that is, $b_M^*>0$. Therefore, we can conclude that $b_M^*>b_N^*$ for $\theta<\theta^*$, and $b_M^*<b_N^*$ for $\theta>\theta^*$.

Proof of Corollary 4.1. According to Table 4.1, when $\psi<\frac{1}{3}$, $\frac{\partial}{\partial\psi}\pi_N^*\left(\psi,\theta\right)=\frac{[(1+\theta)(1-\theta)+\theta(3\theta-1)\psi]f(\psi,\theta)}{(1-\theta)[3(1-\theta)(1-\psi)+4\theta\psi]^3}$, where $f\left(\psi,\theta\right)$ is a polynomial function of ψ and θ with degree of 5 $(\psi^2\theta^3)$. $\frac{\partial}{\partial\psi}f\left(\psi,\theta\right)=-\theta\left(1-3\theta\right)\left(3-7\theta\right)\psi-\left(1-\theta\right)\left(20\theta^2-13\theta+3\right)$. Within the given parameter region, $\frac{\partial}{\partial\psi}f\left(\psi,\theta\right)<0$. Notice that $f\left(\frac{1}{3},\theta\right)=\frac{2}{9}\left(3-\theta\right)\left(12\theta^2-17\theta+3\right)$. When $\theta<\frac{17-\sqrt{145}}{24}$, $f\left(\frac{1}{3},\theta\right)>0$, which implies $f\left(\psi,\theta\right)>0$ for $\forall\psi\in(0,\frac{1}{3})$. Therefore, $\frac{\partial}{\partial\psi}\pi_N^*\left(\psi,\theta\right)>0$, and thus $\pi_N^*\left(\psi_1,\theta\right)-\pi_N^*\left(\psi_2,\theta\right)<0$ and $b_N^*=0$.

Proof of Proposition 4.2. We let $\Delta\hat{\pi}_i=\hat{\pi}_i^1-\hat{\pi}_i^2$, $i\in\{M,N\}$. When $\frac{1}{3}\leq\psi<1$, according to Table 4.2, $\Delta\hat{\pi}_M-\Delta\hat{\pi}_N=\frac{(1-2\theta)}{4(3+\psi)^2(1-\theta)}F\left(\psi,\theta\right)$, where

$$F\left(\psi,\theta\right)=-4\left(\psi^3+3\psi^2+5\psi-1\right)\theta+\left(5\psi^2+17\psi^2+19\psi+7\right).$$

Since $F\left(\psi,0\right)>0$, $F\left(\psi,\frac{1}{2}\right)>0$, and $F\left(\psi,\theta\right)$ is linear in θ, we can conclude that $F\left(\psi,\theta\right)>0$ and hence $\Delta\hat{\pi}_M-\Delta\hat{\pi}_N>0$ for $\forall\theta\in(0,\frac{1}{2})$.

When $0<\psi<\frac{1}{3}$, similarly, $\Delta\hat{\pi}_M-\Delta\hat{\pi}_N=\frac{\psi(1-2\theta)}{(1-\theta)(3+\psi)^2(3-3\psi-3\theta+7\theta\psi)^2}G\left(\psi,\theta\right)$, where $G\left(\psi,\theta\right)$ is a polynomial function of ψ and θ with degree of 7 $(\psi^4\theta^3)$. As it can be verified that $G\left(\psi,\frac{1}{2}\right)>0$ and $\frac{\partial}{\partial\theta}G\left(\psi,\theta\right)<0$ for $\forall\theta\in(0,\frac{1}{2})$, $G\left(\psi,\theta\right)>0$ and thus $\Delta\hat{\pi}_M-\Delta\hat{\pi}_N>0$ for $\forall\theta\in(0,\frac{1}{2})$.

We can further check that $\hat{b}_M=[\Delta\hat{\pi}_M]^+>0$. Therefore, $\hat{b}_M>\hat{b}_N$ for $\forall\psi\in(0,1)$ and $\forall\theta\in(0,\frac{1}{2})$.

Proof of Corollary 4.2. (i) When $\psi<\frac{1}{3}$, according to Table 4.2, $\hat{\pi}_N^1-\hat{\pi}_N^2$ can be simplified as $\frac{\psi F(\psi,\theta)}{(1-\theta)(3+\psi)^2[3(1-\theta)(1-\psi)+4\theta\psi]^2}$, where $F\left(\psi,\theta\right)$ is a polynomial function of ψ and θ with degree of 8 ($\theta^4\psi^4$). We can show that $\frac{\partial^2}{\partial\theta^2}F\left(\psi,\theta\right)$ is monotonic in θ for $\forall\theta\in(0,\frac{1}{2})$, and both $\frac{\partial^2}{\partial\theta^2}F\left(\psi,0\right)$ and $\frac{\partial^2}{\partial\theta^2}F\left(\psi,\frac{1}{2}\right)$ are negative. Therefore, $\frac{\partial^2}{\partial\theta^2}F\left(\psi,\theta\right)<0$, and thus $\frac{\partial}{\partial\theta}F\left(\psi,\theta\right)$ is decreasing in θ. We can further verify that $\frac{\partial}{\partial\theta}F\left(\psi,\frac{1}{2}\right)>0$ and thus $\frac{\partial}{\partial\theta}F\left(\psi,\theta\right)>0$ for $\forall\theta\in(0,\frac{1}{2})$. We also notice that $F\left(\frac{1}{3},\frac{5\sqrt{6}}{6}-2\right)=0$ and $F\left(\psi,\frac{5\sqrt{6}}{6}-2\right)$ is decreasing in ψ, which leads to $F\left(\psi,\frac{5\sqrt{6}}{6}-2\right)>0$ for $\forall\psi\in(0,\frac{1}{3})$. Therefore, $F\left(\psi,\theta\right)>0$ and $\hat{\pi}_N^1>\hat{\pi}_N^2$ for $\forall\theta\in(\frac{5\sqrt{6}}{6}-2,\frac{1}{2})$ and $\forall\psi\in(0,\frac{1}{3})$.

(ii) When $\psi\geq\frac{1}{3}$, $\hat{\pi}_N^2$ is decreasing in ψ. If we view $\hat{\pi}_N^1$ and $\hat{\pi}_N^2$ as functions of ψ, it is sufficient to show $\hat{\pi}_N^1\left(\psi\right)>\hat{\pi}_N^2\left(\frac{1}{3}\right)$ for $\forall\psi\geq\frac{1}{3}$. Notice that $\hat{\pi}_N^1\left(\psi\right)-\hat{\pi}_N^2\left(\frac{1}{3}\right)=\frac{G(\psi,\theta)}{6(1-\theta)(3+\psi)^2}$, where $G\left(\psi,\theta\right)=6\left(1-\psi\right)^2\theta^2+12\left(1-\psi^2\right)\theta+\left(5\psi^2+6\psi-3\right)$. Since $\frac{\partial G}{\partial\theta}>0$, $\frac{\partial G}{\partial\psi}>0$, and $G\left(\frac{1}{3},\frac{5\sqrt{6}}{6}-2\right)=0$, $G\left(\psi,\theta\right)>0$ and thus $\hat{\pi}_N^1\left(\psi\right)>\hat{\pi}_N^2\left(\frac{1}{3}\right)$ for $\forall\psi\in[\frac{1}{3},1)$ and $\forall\theta\in(\frac{5\sqrt{6}}{6}-2,\frac{1}{2})$.

Proof of Proposition 4.3. We plug in $p_M^*\left(\psi,\theta\right)$ and $p_N^*\left(\psi,\theta\right)$ from Eq.(4.5) into Eq.(4.10), and want to show $\frac{\partial}{\partial\psi}W\left(\psi,\theta\right)<0$.

(i) When $\psi<\frac{1}{3}$, $\frac{\partial}{\partial\psi}W\left(\psi,\theta\right)=\frac{F(\psi,\theta)}{2(1-\theta)[3(1-\theta)(1-\psi)+4\theta\psi]^3}$, where $F\left(\psi,\theta\right)$ is a polynomial function of ψ and θ with degree of 8 ($\theta^5\psi^3$). By its concavity ($\frac{\partial^2}{\partial\psi^2}F\left(\psi,\theta\right)<0$), we can show that $F\left(\psi,\theta\right)<0$ for $\forall\psi\in(0,1)$. Therefore, $\frac{\partial}{\partial\psi}W\left(\psi,\theta\right)<0$, for $\forall\psi\in(0,\frac{1}{3})$ and $\forall\theta\in(0,\frac{1}{2})$.

(ii) When $\psi\geq\frac{1}{3}$, $\frac{\partial}{\partial\psi}W\left(\psi,\theta\right)=\frac{12\theta^2-12\theta+1}{8(1-\theta)}$, which is negative when $\theta>\frac{1}{2}-\frac{\sqrt{6}}{6}$. When $\theta<\frac{1}{2}-\frac{\sqrt{6}}{6}$, we can check that M outbids N in both cases, and therefore $W^B=W^C$.

Altogether, according to Propositions 4.1 and 4.2, when $\theta>\theta^*\left(\psi_1,\psi_2\right)$, $W^C=W\left(\psi_1,\theta\right)>W\left(\psi_2,\theta\right)=W^B$; when $\theta<\theta^*\left(\psi_1,\psi_2\right)$, $W^C=W^B=W\left(\psi_2,\theta\right)$.

Proof of Proposition 4.4. (i) When $\psi < \frac{1}{3}$, $D_M(\psi) = \frac{[(2-\theta)+\theta\psi][(1-\theta)-(1-2\theta)\psi]}{3(1-\theta)(1-\psi)+4\theta\psi}$ and $D_N(\psi) = \frac{[(1+\theta)(1-\theta)+\theta(3\theta-1)\psi](1-\psi)}{3(1-\theta)(1-\psi)+4\theta\psi}$. By Eq.(4.12), we have $\frac{\partial}{\partial\psi}G(\psi) = \frac{\theta\cdot F(\psi,\theta)}{(-3+3\theta+3\psi-5\theta\psi-\theta^2\psi+\theta^2\psi^2)^2}$, where $F(\psi,\theta)$ is a quadratic function of ψ and can be proved positive. (If we write $F(\psi,\theta)$ as $A\psi^2+B\psi+C$, we can show $A>0$ and $B^2-4AC<0$.) Therefore, $\frac{\partial}{\partial\psi}G(\psi)>0$, for $\forall\psi \in \left(0,\frac{1}{3}\right)$ and $\forall\theta \in (0,\frac{1}{2})$. (ii) When $\psi \geq \frac{1}{3}$, $D_M(\psi) = \left(\frac{1}{2}-\theta\right)\psi+\frac{1}{2}$ and $D_N(\psi) = \frac{1}{2}-\frac{\psi}{2}$. Therefore, $G(\psi) = \frac{(1-\theta)\psi}{2(1-\theta\psi)}$, which is increasing in ψ. Altogether, $\frac{\partial}{\partial\psi}G(\psi) > 0$ for $\forall\theta \in (0,\frac{1}{2})$ and thus, according to Propositions 4.1 and 4.2, $G^C = G(\psi_1) < G(\psi_2) = G^B$ when $\theta > \theta^*(\psi_1,\psi_2)$, and $G^C = G^B = G(\psi_2)$ when $\theta < \theta^*(\psi_1,\psi_2)$.

Proof of Lemma 4.2. Recall that $\hat{b}_N = [\hat{\pi}_N^1 - \hat{\pi}_N^2]^+$, $b_N^* = [\pi_N^*(\psi_1,\theta) - \pi_N^*(\psi_2,\theta)]^+$, and $\hat{\pi}_N^2 = \pi_N^*(\psi_2,\theta)$. By Table 4.2, $\hat{\pi}_N^1$ is increasing in ψ (by simply checking the first-order derivative) and thus $\hat{\pi}_N^1(\psi_2) > \hat{\pi}_N^1(\psi_1)$ for all $0 < \psi_1 < \psi_2 < 1$. From Corollary 4.2, $\hat{\pi}_N^1(\psi) > \hat{\pi}_N^2(\psi)$ for *any* $\psi \in (0,1)$ when $\theta \in (\frac{5\sqrt{6}}{6}-2,\frac{1}{2})$. Therefore, $\hat{\pi}_N^1(\psi_2) > \hat{\pi}_N^2(\psi_1)$. As $\hat{\pi}_N^2(\psi_1) = \pi_N^*(\psi_1,\theta)$ (due to the same information structure), $\hat{\pi}_N^1(\psi_2) - \hat{\pi}_N^2 > \pi_N^*(\psi_1,\theta) - \pi_N^*(\psi_2,\theta)$. When $\theta \in (\frac{5\sqrt{6}}{6}-2,\frac{1}{2})$, since $\hat{b}_N > 0$ by Corollary 4.2, $IR^B = \hat{b}_N > b_N^* \geq \min\{b_M^*, b_N^*\} = IR^C$ for all $0 < \psi_1 < \psi_2 < 1$.

Proof of Lemma 4.3. We plug in $p_M^*(\psi,\theta)$ and $p_N^*(\psi,\theta)$ from Eq.(4.5) into Eq.(4.13), and want to show $\frac{\partial}{\partial\psi}CS(\psi,\theta) < 0$. (i) When $\psi < \frac{1}{3}$, $\frac{\partial}{\partial\psi}CS(\psi,\theta) = \frac{(1-2\theta)F(\psi,\theta)}{2(1-\theta)[3(1-\theta)(1-\psi)+4\theta\psi]^3}$, where $F(\psi,\theta)$ is a polynomial function of ψ and θ with degree of 7 ($\theta^4\psi^3$). As we can show, for $\forall\psi \in \left(0,\frac{1}{3}\right)$, $\frac{\partial}{\partial\psi}F(\psi,\theta) > 0$ when $\theta < \frac{3}{7}$, and $\frac{\partial^2}{\partial\psi^2}F(\psi,\theta) > 0$ when $\theta > \frac{3}{7}$. In other words, $F(\psi,\theta)$ is either increasing or convex in ψ. Notice that $F(0,\theta) = (1-\theta)^2(-33-8\theta+22\theta^2) < 0$ and $F\left(\frac{1}{3},\theta\right) = -\frac{2}{27}(3-\theta)^2(33-37\theta+10\theta^2) < 0$ for $\forall\theta \in \left(0,\frac{1}{2}\right)$, we can conclude that $F(\psi,\theta) < 0$ and hence $CS(\psi,\theta)$ is decreasing in ψ for $\forall\theta \in \left(0,\frac{1}{2}\right)$, $\forall\psi \in \left(0,\frac{1}{3}\right)$. (ii) When $\psi \geq \frac{1}{3}$, we have $CS(\psi,\theta) = \frac{(1-2\theta)(1+2\theta)}{8(1-\theta)}(1-\psi)$, which is decreasing in ψ. Altogether, according to Propositions 4.1 and 4.2, $CS^C = CS(\psi_1,\theta) > CS(\psi_2,\theta) = CS^B$ when $\theta > \theta^*(\psi_1,\psi_2)$, and $CS^C = CS^B = CS(\psi_2,\theta)$ when $\theta < \theta^*(\psi_1,\psi_2)$.

Proof of Proposition 4.5. By Lemma 4.3, $CS^C > CS^B$ when $\theta > \theta^*(\psi_1,\psi_2)$. Since $LG = g(CS)$ is strictly increasing in CS, $LG^C > LG^B$ when $\theta > \theta^*(\psi_1,\psi_2)$.

Combining the result from Lemma 4.2, we have $IR^C < IR^B$ and $LG^C > LG^B$ when $\theta > \max\{\theta^*(\psi_1, \psi_2), \frac{5\sqrt{6}}{6} - 2\}$.

Proof of Proposition 4.6. We first derive the second-stage equilibrium prices. When M wins the first sponsored slot, for $\psi_2 < \frac{1}{3(1-\lambda)}$,

$$\begin{cases} \tilde{p}_M^1 = & \frac{(1-\lambda)\theta\psi_2+2-\theta}{4(1-\lambda)\psi_2\theta+3(1-\lambda)(1-\psi_2)(1-\theta)+3\lambda(1-\theta)} \\ \tilde{p}_N^2 = & \frac{(1-\lambda)\psi_2\theta(3\theta-1)+1-\theta^2}{(1-\theta)[4(1-\lambda)\psi_2\theta+3(1-\lambda)(1-\psi_2)(1-\theta)+3\lambda(1-\theta)]}. \end{cases} \tag{D.1}$$

When N wins the first sponsored slot,

$$\begin{cases} \tilde{p}_M^2 = & \frac{\lambda\psi_2(1-\theta)+2-\theta}{(1-\theta)(\lambda\psi_2+3)} \\ \tilde{p}_N^1 = & \frac{\lambda\psi_2(1-\theta)+1+\theta}{(1-\theta)(\lambda\psi_2+3)}. \end{cases} \tag{D.2}$$

(i) We can calculate N's equilibrium profit in both cases: $\tilde{\pi}_N^1 = \tilde{p}_N^1 D_N^1(\tilde{p}_N^1, \tilde{p}_M^2)$ and $\tilde{\pi}_N^2 = \tilde{p}_N^2 D_N^2(\tilde{p}_N^2, \tilde{p}_M^1)$ according to Eq.(4.18) and Eq.(4.17). We then have $\Delta\tilde{\pi}_N(\psi_2, \theta, \lambda) = \tilde{\pi}_N^1 - \tilde{\pi}_N^2$, and $\frac{\partial}{\partial\lambda}\Delta\tilde{\pi}_N(\psi_2, \theta, \lambda)|_{\lambda=0} = \frac{\psi_2 F(\psi_2,\theta)}{27(1-\theta)[4\psi_2\theta+3(1-\psi_2)(1-\theta)]}$, where $F(\psi_2, \theta)$ is a polynomial function of ψ_2 and θ with degree of 8 ($\theta^5\psi_2^3$). We can show that $\frac{\partial^2}{\partial\psi_2^2}F(\psi_2, \theta) = f(\theta)[4\psi_2\theta + 3(1-\psi_2)(1-\theta)]$, where $f(\theta)$ is some expression that only contains θ. For a given θ, $\frac{\partial^2}{\partial\psi_2^2}F(\psi_2, \theta)$ does not change sign for $\forall\psi_2 \in (0, 1)$; that is, $\frac{\partial}{\partial\psi_2}F(\psi_2, \theta)$ is monotonic in ψ_2. We can check that $\frac{\partial}{\partial\psi_2}F(0, \theta) < 0$ and $\frac{\partial}{\partial\psi_2}F(1, \theta) < 0$, $\forall\theta \in (0, \frac{1}{2})$. Therefore, $\frac{\partial}{\partial\psi_2}F(\psi_2, \theta) < 0$ for $\forall\psi_2 \in (0, 1)$ and $\forall\theta \in (0, \frac{1}{2})$. Since we focus on the case where $\psi_2 < \frac{1}{3(1-\lambda)}$ ($= \frac{1}{3}$ at $\lambda = 0$) and we notice that $F\left(\frac{1}{3}, \theta\right) = \frac{2}{27}(3-\theta)^2(32\theta^3 + 244\theta^2 - 523\theta + 129)$ crosses zero from above once when changing θ at $[0, 1]$, $F\left(\frac{1}{3}, \theta\right) > 0$ when $\theta \in (0, \theta_0)$. By monotonicity of F, $F(\psi_2, \theta) > 0$ and thus $\frac{\partial}{\partial\lambda}\Delta\tilde{\pi}_N(\psi_2, \theta, \lambda)|_{\lambda=0} > 0$, for $\forall\psi_2 \in (0, \frac{1}{3(1-\lambda)})$ and $\forall\theta \in (0, \theta_0)$. We can further check that $\theta_0 < \theta^*(0, \psi_2)$ (as defined by Eq.(4.7)) for the given parameter region, which validates the presumed bidding outcome (i.e., $b_N < b_M$).

(ii) Substituting equilibrium prices in Eq.(D.1) into the expression of equilibrium consumer surplus in Eq.(4.19), we can derive $\frac{\partial}{\partial\lambda}\tilde{CS}(\psi_2, \theta, \lambda)|_{\lambda=0} = \frac{(1-2\theta)\psi_2 G(\psi_2,\theta)}{2(1-\theta)[4\psi_2\theta+3(1-\psi_2)(1-\theta)]^3}$, where $G(\psi_2, \theta)$ is a polynomial function of ψ_2 and θ with degree of 7 ($\theta^4\psi_2^3$). As we can show, for $\forall\psi_2 \in \left(0, \frac{1}{3(1-\lambda)}\right)$, $G(\psi_2, \theta)$ is decreasing in ψ_2 when $\theta < \frac{3}{7}$, and $G(\psi_2, \theta)$ is concave in ψ_2 when $\theta > \frac{3}{7}$. Notice that $G(0, \theta) = (1-\theta)^2(33 + 8\theta - 22\theta^2) > 0$ and

$G\left(\frac{1}{3},\theta\right)=\frac{2}{27}(3-\theta)^2(33-37\theta+10\theta^2)>0$, we can conclude that $G\left(\psi_2,\theta\right)>0$ and hence $\frac{\partial}{\partial\lambda}\tilde{CS}\left(\psi_2,\theta,\lambda\right)|_{\lambda=0}>0$ for $\forall\psi_2\in\left(0,\frac{1}{3(1-\lambda)}\right)$ and $\forall\theta\in(0,\theta_0)$.

(iii) Substituting equilibrium prices in Eq.(D.1) into the welfare function in Eq.(4.20), we can derive $\frac{\partial}{\partial\lambda}\tilde{W}\left(\psi_2,\theta,\lambda\right)|_{\lambda=0}=\frac{\psi_2 H(\psi_2,\theta)}{2(1-\theta)[4\psi_2\theta+3(1-\psi_2)(1-\theta)]^3}$, where $H\left(\psi_2,\theta\right)$ is a polynomial function of ψ_2 and θ with degree of 8 $(\theta^5\psi_2^3)$. Following similar arguments as in (i), we can show that $H\left(\psi_2,\theta\right)$ is decreasing in ψ_2 for $\forall\psi_2\in(0,1)$ and $\forall\theta\in(0,\theta_0)$. Furthermore, $H\left(\frac{1}{3},\theta\right)=\frac{2}{27}\left(3-\theta\right)^2\left[\theta\left(12\left(\theta-\frac{3}{2}\right)^2-8\right)+3\right]>0$ for $0<\theta<\frac{1}{2}$. Therefore, $H\left(\psi_2,\theta\right)>0$, and thus $\frac{\partial}{\partial\lambda}\tilde{W}\left(\psi_2,\theta,\lambda\right)|_{\lambda=0}>0$ for $\forall\psi_2\in(0,\frac{1}{3(1-\lambda)})$ and $\forall\theta\in(0,\theta_0)$.

(iv) Substituting equilibrium prices in Eq.(D.1) into the expression of sales Gini coefficient in Eq.(4.21), we can derive $\frac{\partial}{\partial\lambda}\tilde{G}\left(\psi_2,\theta,\lambda\right)|_{\lambda=0}=\frac{\theta\psi_2\left(A\psi_2^2+B\psi_2+C\right)}{\left(\theta^2\psi_2^2-\theta^2\psi_2-5\theta\psi_2+3\theta+3\psi_2-3\right)^2}$, where $A=\theta^3-15\theta^2+13\theta-3$, $B=-2\theta^3+18\theta^2-22\theta+6$, and $C=\theta^3-7\theta^2+11\theta-5$. As we can verify that $B^2-4AC<0$ and $A<0$, $\frac{\partial}{\partial\lambda}\tilde{G}\left(\psi_2,\theta,\lambda\right)|_{\lambda=0}<0$ for $\forall\psi_2\in(0,\frac{1}{3(1-\lambda)})$ and $\forall\theta\in(0,\theta_0)$.

Appendix E

More Results for Chapter 4

In this section, we extend the model from duopolistic to oligopolistic competition and show that the qualitative results and the main implications continue to hold.

E.1 Model

We now consider $n\,(\geq 2)$ firms in the market. One of them sell products catering to the mainstream market, which is termed as M; the other $n-1$ firms' products are designed for particular niche markets, which are denoted as $N_1, ..., N_{n-1}$. There is a continuum of consumers with mass 1. Each consumer has a unit demand of the product. Consumers have different preferences. Following the main model, we assume θ $(0 < \theta < \frac{1}{2})$ of them are N-type consumers who prefer the niche firms' products to the mainstream firm's, and $1-\theta$ are M-type consumers who prefer the mainstream product. Among the $n-1$ niche firms, we consider the case in which niche firms develop particular features of the product that are mutually exclusive so that each consumer may accept only one niche firm's product and derives zero utility for the others. For ease of exposition, we assume all niche firms equally share the market preference. An N-type consumer derives utility v from consuming the particular type of the niche product that she can accept, and derives discounted utility $\tilde{k}v$ from consuming the mainstream product. An M-type consumer derives utility v from consuming the mainstream product and discounted utility $\tilde{k}v$ from consuming the niche product that she can accept. $\tilde{k}$ is uniformly distributed between 0 and 1 among all consumers. We normalize v to 1 without loss of generality.

We follow a similar way in modeling consumers' click behavior on the search engine results page. The M firm is listed in a top organic position which attracts most attention (i.e., α_{i_M} can be very high). Similar as before, we let $\alpha_{i_M} = 1$ to emphasize the diminishing promotive incentive when an advertiser's organic rank is high. In other words, consumers click M's link with probability 1. In contrast, the niche firms' organic ranks are much less satisfactory and the differences among their organic exposures can be negligible compared to the difference between the mainstream's and theirs. We thus assume that the niche firms' overall attention levels are determined by their sponsored ranking, that is, $1 - \left(1-\alpha_{i_{N_k}}\right)(1-\beta_j)\left(1-\gamma_{i_{N_k}j}\right) = 1 - \left(1-\alpha_{i_{N_{k'}}}\right)(1-\beta_j)\left(1-\gamma_{i_{N_{k'}}j}\right)$ for any $k, k' \in \{1, ..., n-1\}$. More specifically, for a niche firm winning the jth sponsored slot, the combined probability of its link (either organic or sponsored) being clicked is $1-\psi_j$. There are $m\,(\geq 2)$ sponsored slots and we only discuss the case in which $m \geq n$. Similar analysis can easily be applied to the case of $m < n$. Without loss of generality, we let the exposure of sponsored positions monotonically decrease from the first to the last, that is, $0 < \psi_1 < ... < \psi_n < 1$.

E.2 Analysis and Results

We can derive the equilibrium pricing similarly as in the main model. The demand function facing the niche firm staying in the jth sponsored position, given M's price p_M, can be written as

$$D_N^j(p; p_M) = \frac{1}{n-1}\left(1-\psi_j\right) S_N\left(p, p_M\right),$$

where S_N is defined by Eq.(4.1). Maximizing the profit function $pD_N^j(p; p_M)$ gives the optimal pricing. Notice that the maximization problem is actually independent of ψ_j, which implies that all niche firms charge the same price in equilibrium. Given niche firms' price p_N, the demand function facing M when it stays in the kth sponsored position can be written as

$$\begin{aligned} D_M^k(p; p_N) &= \sum_{i \neq k} \frac{1}{n-1}\left[\psi_i A_M(p) + (1-\psi_i) S_M(p, p_N)\right] \\ &= \bar{\psi}_{-k} A_M(p) + \left(1-\bar{\psi}_{-k}\right) S_M(p, p_N), \end{aligned}$$

where $\bar{\psi}_{-k} = \frac{1}{n-1}\sum_{i \neq k} \psi_i$. Maximizing the profit functions simultaneously, we can derive the equilibrium prices (when M stays in the kth sponsored position)

$$\begin{cases} p_M^* & = \min\{\frac{2-\theta\left(1-\bar{\psi}_{-k}\right)}{3(1-\theta)\left(1-\bar{\psi}_{-k}\right)+4\theta\bar{\psi}_{-k}}, 1\} \\ p_N^* & = \frac{\theta+(1-\theta)p_M^*}{2(1-\theta)}. \end{cases}$$

Therefore, the equilibrium profit functions can be derived accordingly as follows.

$$\begin{cases} \pi_M^k = \pi_M^*\left(\bar{\psi}_{-k}, \theta\right) \\ \pi_N^{j(k)} = \frac{1}{n-1}\left(1-\psi_j\right) f\left(\bar{\psi}_{-k}, \theta\right), \end{cases}$$

where π_M^k is the equilibrium profit for the M firm if it stays in the kth sponsored position, and $\pi_N^{j(k)}$ stands for the equilibrium profit of the N firm staying in the jth sponsored position when M gets the kth sponsored slot ($j \neq k$). Here, $\pi_M^*(\psi, \theta)$ is the same as is defined in Table 4.1, while $f(\psi, \theta)$ is defined as

$$f(\psi, \theta) = \begin{cases} \frac{\left[1-\theta^2+\theta(3\theta-1)\psi\right]^2}{(1-\theta)[3(1-\theta)(1-\psi)+4\theta\psi]^2} & \psi < \frac{1}{3} \\ \frac{1}{4(1-\theta)} & \psi \geq \frac{1}{3}. \end{cases}$$

Before we derive the bidding equilibrium, we first summarize a result which will be useful in the analysis of equilibrium bidding. In fact, the following result is a counterpart of Lemma 4.1 in the oligopolistic case. It shows that the difference between the mainstream firm's and the niche firms' incentives to improve their sponsored ranks decrease as the competence difference reduces (i.e., as θ increases).

Lemma E.1. $\pi_M^*\left(\psi', \theta\right) - \pi_M^*\left(\psi''', \theta\right) - \frac{1}{n-1}\left(1-x_1\right) f\left(\psi'', \theta\right) + \frac{1}{n-1}\left(1-x_2\right) f\left(\psi', \theta\right)$ is decreasing in θ, where $1 > \psi' > \psi'' \geq \psi''' > 0$ and $1 > x_2 \geq x_1 > 0$.

To derive the bidding equilibrium, we follow Edelman et al. (2007) and consider the locally envy-free equilibrium in the generalized second-price auctions. In a locally envy-free equilibrium, any advertiser does not want to exchange bids with the one ranked one position above it in the sponsored list. We focus on the particular type of locally envy-free equilibrium studied by Edelman et al. (2007), in which each advertiser bids an amount equals its own payment plus its own value difference between staying in the current position and moving one position up. In other words, each advertiser's payment is equal to the negative externality that it imposes on all the other advertisers. The equilibrium analysis is more complex in our setting because any change of M's position will change the values of all niche firms in all positions.

We are particularly interested in two types of equilibria, namely, when the mainstream firm wins the first sponsored position and the niche firms stay in the second to the nth sponsored positions, and when the niche firms stay in the first $n-1$ sponsored positions while the mainstream firm gets the last. Applying the results from Lemma E.1, we show that when θ is large, the latter may hold in equilibrium, and the former can be an equilibrium when θ is small, which are very similar to Proposition 4.1.

Proposition E.1. *Generically, there exist cutoffs $\hat{\theta}(\psi_1, ..., \psi_n)$ and $\tilde{\theta}(\psi_1, ..., \psi_n)$, such that when $0 < \theta < \hat{\theta}(\psi_1, ..., \psi_n)$, M winning the first sponsored position and all niche firms staying in the second to the nth positions is an equilibrium, and when $\tilde{\theta}(\psi_1, ..., \psi_n) < \theta < \frac{1}{2}$, all niche firms staying in the first $n-1$ positions and M staying in the last is an equilibrium.*

Proposition E.1 reveals a similar pattern in the equilibrium bidding outcomes. With reasonable market preference shares, although still weaker than the leading firm, niche firms have higher bidding incentives and get better sponsored positions. Such outcome, which is not aligned with advertisers' inherent competitive strength, reflects the interplay between organic listing and sponsored bidding, and the balance between the promotive and preventive effects. On the other hand, when niche firms are too weak, the leading firm's preventive motivation dominates, making it occupy the top sponsored position.

Given the similarity in the equilibrium bidding outcomes, the rest analysis and the main results from the duopoly case can be expected to hold qualitatively when extended to the oligopoly case.

E.3 Proofs

Proof of Lemma E.1. To prove that the objective function is decreasing in θ is to prove

$$\frac{\partial}{\partial\theta}\pi_M^*\left(\psi',\theta\right)-\frac{\partial}{\partial\theta}\pi_M^*\left(\psi''',\theta\right)+\frac{1}{n-1}\left(1-x_2\right)\frac{\partial}{\partial\theta}f\left(\psi',\theta\right)-\frac{1}{n-1}\left(1-x_1\right)\frac{\partial}{\partial\theta}f\left(\psi'',\theta\right)<0. \tag{E.1}$$

Recall that Lemma 4.1 shows that $\frac{\partial^2}{\partial\psi\partial\theta}\pi_M^*\left(\psi,\theta\right)<0$. Since $\psi'>\psi'''$, therefore, $\frac{\partial}{\partial\theta}\pi_M^*\left(\psi',\theta\right)-\frac{\partial}{\partial\theta}\pi_M^*\left(\psi''',\theta\right)<0$, that is, the first half of the LHS of Eq.(E.1) is negative. Also, we can show that when $\theta\geq 0.153$, $\frac{\partial^2}{\partial\psi\partial\theta}f\left(\psi,\theta\right)\leq 0$. Notice that $\frac{\partial}{\partial\theta}f\left(\psi,\theta\right)>0$. Therefore,

$$\left(1-x_2\right)\frac{\partial}{\partial\theta}f\left(\psi',\theta\right)-\left(1-x_1\right)\frac{\partial}{\partial\theta}f\left(\psi'',\theta\right)\leq\left(1-x_1\right)\left[\frac{\partial}{\partial\theta}f\left(\psi',\theta\right)-\frac{\partial}{\partial\theta}f\left(\psi'',\theta\right)\right]\leq 0.$$

Thus, when $\theta\geq 0.153$, the second half of the LHS of Eq.(E.1) is also non-positive, which implies that Eq.(E.1) holds.

When $\theta<0.153$, the second half of the LHS of Eq.(E.1) could be positive so that we need to compare the magnitudes of the two parts. We discuss the case in which $\psi'''\leq\psi''<\psi'\leq\frac{1}{3}$, the other cases can be easily proved. By mean value theorem, we have

$$\frac{\partial}{\partial\theta}\pi_M^*\left(\psi',\theta\right)-\frac{\partial}{\partial\theta}\pi_M^*\left(\psi''',\theta\right)=\frac{\partial^2}{\partial\psi\partial\theta}\pi_M^*\left(\hat{\psi},\theta\right)\left(\psi'-\psi'''\right),$$

where $\hat{\psi}$ is some value between ψ' and ψ'''. Similarly,

$$\begin{aligned}
&\frac{1}{n-1}\left(1-x_2\right)\frac{\partial}{\partial\theta}f\left(\psi',\theta\right)-\frac{1}{n-1}\left(1-x_1\right)\frac{\partial}{\partial\theta}f\left(\psi'',\theta\right)\\
\leq\;&\frac{1}{n-1}\left(1-x_1\right)\left[\frac{\partial}{\partial\theta}f\left(\psi',\theta\right)-\frac{\partial}{\partial\theta}f\left(\psi'',\theta\right)\right]\\
=\;&\frac{1}{n-1}\left(1-x_1\right)\frac{\partial^2}{\partial\psi\partial\theta}f\left(\tilde{\psi},\theta\right)\left(\psi'-\psi''\right),
\end{aligned}$$

where $\tilde{\psi}$ is some value between ψ' and ψ''. Since $\psi'-\psi'''\geq\psi'-\psi''$, we can conclude that Eq.(E.1) holds if we can show $-\frac{\partial^2}{\partial\psi\partial\theta}\pi_M^*\left(\hat{\psi},\theta\right)>\frac{\partial^2}{\partial\psi\partial\theta}f\left(\tilde{\psi},\theta\right)$. As we can verify, both $\frac{\partial^2}{\partial\psi\partial\theta}\pi_M^*\left(\psi,\theta\right)$ and $\frac{\partial^2}{\partial\psi\partial\theta}f\left(\psi,\theta\right)$ are decreasing in ψ for $\forall\psi\in\left(0,\frac{1}{3}\right)$ when $\theta<0.153$. Therefore,

$$\begin{aligned}
\frac{\partial^2}{\partial\psi\partial\theta}\pi_M^*\left(\hat{\psi},\theta\right)+\frac{\partial^2}{\partial\psi\partial\theta}f\left(\tilde{\psi},\theta\right)\;&<\;\frac{\partial^2}{\partial\psi\partial\theta}\pi_M^*\left(0,\theta\right)+\frac{\partial^2}{\partial\psi\partial\theta}f\left(0,\theta\right)\\
&=\;-\frac{2\left(2+13\theta-3\theta^2+\theta^3\right)}{27\left(1-\theta\right)^3}\\
&<\;0.
\end{aligned}$$

Altogether, we have shown that Eq.(E.1) holds for $\forall\theta \in \left(0, \frac{1}{2}\right)$, which means the objective function is decreasing in θ.

Proof of Proposition E.1. (i) Consider the locally envy-free equilibrium in which the $n-1$ niche firms' bids (labeled such that $b_N^n < b_N^{n-1} < ... < b_N^2$) are

$$\begin{cases} b_N^n = \pi_N^{n-1(1)} - \pi_N^{n(1)} & \\ b_N^i = \pi_N^{i-1(1)} - \pi_N^{i(1)} + b_N^{i+1}, & i = 3, ..., n-1 \\ b_N^2 = \max\{\pi_N^{1(2)} - \pi_N^{2(1)}, 0\} + b_N^3 & \end{cases}$$

and the mainstream firm bids any amount greater than b_N^2. As a result, M wins the first position and niche firms stay in the 2nd through the nth positions.

Now we investigate under what conditions the above bidding strategy profile is indeed an equilibrium. It is easy to see that all niche firms have no profitable deviations. To ensure no profitable deviation of M, $\pi_M^1 - b_N^2 \geq \pi_M^k - b_N^{k+1}$ has to be satisfied for all $k = 2, ..., n$ (Let $b_N^{n+1} = 0$). In other words, the following conditions have to be satisfied.

$$\pi_M^1 - \left(\max\{\pi_N^{1(2)}, \pi_N^{2(1)}\} - \pi_N^{n(1)}\right) \geq \pi_M^k - \left(\pi_N^{k(1)} - \pi_N^{n(1)}\right), k = 2, ..., n \quad ;$$

or equivalently,

$$\begin{cases} \pi_M^1 - \pi_N^{1(2)} \geq \pi_M^k - \pi_N^{k(1)} & k = 2, ..., n \\ \pi_M^1 - \pi_N^{2(1)} \geq \pi_M^k - \pi_N^{k(1)} & k = 2, ..., n \end{cases},$$

which are further equivalent to that

$$\begin{cases} \pi_M^* \left(\bar{\psi}_{-1}, \theta\right) - \pi_M^* \left(\bar{\psi}_{-k}, \theta\right) + \frac{1}{n-1} \left(1 - \psi_k\right) f \left(\bar{\psi}_{-1}, \theta\right) - \frac{1}{n-1} \left(1 - \psi_1\right) f \left(\bar{\psi}_{-2}, \theta\right) \geq 0 \\ \pi_M^* \left(\bar{\psi}_{-1}, \theta\right) - \pi_M^* \left(\bar{\psi}_{-k}, \theta\right) + \frac{1}{n-1} \left(1 - \psi_k\right) f \left(\bar{\psi}_{-1}, \theta\right) - \frac{1}{n-1} \left(1 - \psi_2\right) f \left(\bar{\psi}_{-1}, \theta\right) \geq 0 \end{cases}$$

holds for all $k = 2, ..., n$. By Lemma E.1, the LHS of the first inequality is decreasing in θ. Also, it is easy to show that the LHS of the second inequality is decreasing in θ, given that $\frac{\partial}{\partial\theta} f\left(\psi, \theta\right) > 0$. As a result, as long as the values of $\{\psi_i\}_{i=1}^n$ satisfy

$$\begin{cases} \pi_M^* \left(\bar{\psi}_{-1}, 0\right) - \pi_M^* \left(\bar{\psi}_{-k}, 0\right) + \frac{1}{n-1} \left(1 - \psi_k\right) f \left(\bar{\psi}_{-1}, 0\right) - \frac{1}{n-1} \left(1 - \psi_1\right) f \left(\bar{\psi}_{-2}, 0\right) > 0 \\ \pi_M^* \left(\bar{\psi}_{-1}, 0\right) - \pi_M^* \left(\bar{\psi}_{-k}, 0\right) + \frac{1}{n-1} \left(1 - \psi_k\right) f \left(\bar{\psi}_{-1}, 0\right) - \frac{1}{n-1} \left(1 - \psi_2\right) f \left(\bar{\psi}_{-1}, 0\right) > 0 \end{cases}$$

for $k = 2, ..., n$, which is a loose parametric condition that can be satisfied under most values of $\{\psi_i\}_{i=1}^n$, we can conclude that there exists $\hat{\theta}\left(\psi_1, ..., \psi_n\right)$such that when $0 < \theta < \hat{\theta}\left(\psi_1, ..., \psi_n\right)$, the described bidding strategy profile is an equilibrium.

(ii) Similarly, consider the locally envy-free equilibrium in which the $n-1$ niche firms' bids (labeled such that $b_N^{n-1} < b_N^{n-2} < ... < b_N^1$) are

$$\begin{cases} b_N^{n-1} = \pi_N^{n-2(n)} - \pi_N^{n-1(n)} + b_M \\ b_N^i = \pi_N^{i-1(n)} - \pi_N^{i(n)} + b_N^{i+1}, \qquad i = 2, ..., n-2 \\ b_N^1 > b_N^2 \end{cases}$$

and the mainstream firm bids $b_M = \pi_M^{n-1} - \pi_M^n$. As a result, the niche firms stay in the first $n-1$ positions and M stays in the last one.

To see under what conditions the above bidding strategy profile is indeed an equilibrium, we need to ensure that any niche firm does not have profitable deviation, such that $\pi_N^{n(n-1)} \leq \pi_N^{k(n)} - b_N^{k+1} = \pi_N^{n-1(n)} - \left(\pi_M^{n-1} - \pi_M^n\right)$ $(k = 1, ..., n-1)$, as well as that M does not have profitable deviation, such that $\pi_M^k - b_N^k \leq \pi_M^n$ for $k = 2, ..., n-1$. (Note that no deviation to the first position for M can easily hold as long as b_N^1 is high enough.) We can organize these conditions as

$$\begin{cases} \pi_M^{n-1} - \pi_M^n + \pi_N^{n(n-1)} - \pi_N^{n-1(n)} \leq 0 \\ \pi_M^k - \pi_M^{n-1} + \pi_N^{n-1(n)} - \pi_N^{k-1(n)} \leq 0, \quad k = 2, ..., n-1; \end{cases}$$

or equivalently,

$$\begin{cases} \pi_M^*\left(\bar{\psi}_{-(n-1)}, \theta\right) - \pi_M^*\left(\bar{\psi}_{-n}, \theta\right) + \frac{1}{n-1}\left(1-\psi_n\right) f\left(\bar{\psi}_{-(n-1)}, \theta\right) - \frac{1}{n-1}\left(1-\psi_{n-1}\right) f\left(\bar{\psi}_{-n}, \theta\right) \leq 0 \\ \pi_M^*\left(\bar{\psi}_{-k}, \theta\right) - \pi_M^*\left(\bar{\psi}_{-(n-1)}, \theta\right) + \frac{1}{n-1}\left(1-\psi_{n-1}\right) f\left(\bar{\psi}_{-n}, \theta\right) - \frac{1}{n-1}\left(1-\psi_{k-1}\right) f\left(\bar{\psi}_{-n}, \theta\right) \leq 0, \end{cases}$$

where $k = 2, ..., n-1$. By Lemma E.1, the LHS of the first inequality is decreasing in θ. Also, it is easy to show that the LHS of the second inequality is decreasing in θ, given that $\frac{\partial}{\partial\theta} f(\psi, \theta) > 0$. As a result, as long as the values of $\{\psi_i\}_{i=1}^n$ satisfy

$$\begin{cases} \pi_M^*\left(\bar{\psi}_{-(n-1)}, \frac{1}{2}\right) - \pi_M^*\left(\bar{\psi}_{-n}, \frac{1}{2}\right) + \frac{1}{n-1}\left(1-\psi_n\right) f\left(\bar{\psi}_{-(n-1)}, \frac{1}{2}\right) - \frac{1}{n-1}\left(1-\psi_{n-1}\right) f\left(\bar{\psi}_{-n}, \frac{1}{2}\right) < 0 \\ \pi_M^*\left(\bar{\psi}_{-k}, \frac{1}{2}\right) - \pi_M^*\left(\bar{\psi}_{-(n-1)}, \frac{1}{2}\right) + \frac{1}{n-1}\left(1-\psi_{n-1}\right) f\left(\bar{\psi}_{-n}, \frac{1}{2}\right) - \frac{1}{n-1}\left(1-\psi_{k-1}\right) f\left(\bar{\psi}_{-n}, \frac{1}{2}\right) < 0, \end{cases}$$

for $k = 2, ..., n-1$, which is not difficult to be satisfied under most values of $\{\psi_i\}_{i=1}^n$, we can conclude that there exists $\tilde{\theta}(\psi_1, ..., \psi_n)$ such that when $\tilde{\theta}(\psi_1, ..., \psi_n) < \theta < \frac{1}{2}$, the described bidding strategy profile is an equilibrium.

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